Senior 2 Applied Mathematics Exercises

A Supplement to
A Foundation for
Implementation

Renewing Education:
New Directions

Manitoba
Education
and Training
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CONTENTS

Acknowledgements *iii*

Introduction *1*

A: Spreadsheets *A-1*

B: Exploring Mathematics Using Technology *B-1*

C: Technical Communication *C-1*

E: 2D/3D Projects *E-1*

F: Relations and Functions *F-1*

G: Coordinate Geometry *G-1*

H: Measurement Technology *H-1*

I: Trigonometry *I-1*

Note that exercises are **not** provided for the following units:

- D: Linear Models and Patterns

  For exercises on Linear Models and Patterns, refer to Module 4 in *Senior 2 Applied Mathematics: A Full Course for Distance Education Delivery* by Manitoba Education and Training, 1998.

- J: Data Management and Analysis

  For exercises on Data Management and Analysis, refer to Module 10 in *Senior 2 Applied Mathematics: A Full Course for Distance Education Delivery* by Manitoba Education and Training, 1998.

INTRODUCTION

Senior 2 Applied Mathematics includes the following 10 units:

Unit A:	Spreadsheets	9 hrs
Unit B:	Exploring Mathematics Using Technology	9 hrs
Unit C:	Technical Communication	9 hrs
Unit D:	Linear Models and Patterns	13 hrs
Unit E:	2D/3D Geometry	9 hrs
Unit F:	Relations and Functions	13 hrs
Unit G:	Coordinate Geometry	9 hrs
Unit H:	Measurement Technology	13 hrs
Unit I:	Trigonometry	13 hrs
Unit J:	Data Management and Analysis	13 hrs

The arrangement of units is not intended as a detailed sequence of instruction. Some concepts and skills from Units A, B, C, and J would need to be introduced early in the term/semester/year as they are applied throughout. For example, techniques for determining the line of best fit found in Unit J: Data Management and Analysis are refined in Unit B: Exploring Mathematics Using Technology and Unit D: Linear Models and Patterns. So that students understand and can use both metric and imperial units of measure, exercises include both measurement systems. Teachers will need to modify examples with imperial units or introduce early on imperial units found in Unit H: Measurement Technology. Furthermore, most projects and activities integrate concepts and skills from multiple topic areas.

The questions and projects provided in *Senior 2 Applied Mathematics Exercises: A Supplement to A Foundation for Implementation* vary in difficulty. Some require short answers and are meant to be done quickly; others are more project oriented and may require longer periods of time and/or additional materials. Teachers are encouraged to look through the exercises and select the questions or projects that best suit their students' needs.

Cautionary Note

Some of the activities or problems in this document may involve chance and probability. In some families and communities, the connection between probability and gambling may be problematic; for example, parents/guardians may not approve of playing cards, dice, or prize money. As an alternative, activities or problems can be reworded to include numbered index cards, number cubes, and points or credits.

Unit A
Spreadsheets

Note

The Spreadsheet exercises require the templates below. These templates were prepared using ClarisWorks on the Macintosh computer. If students are using other spreadsheet programs, the formulas may need to be adjusted.

Template 1

	A	B
1	Length	
2	Width	
3		
4	Perimeter	=2*(B1+B2)
5	Area	=B1*B2
6	Diagonal Length	=SQRT(B1^2+B2^2)

Template 2

	A	B
1	Reg Hours Worked	40
2	OT Hours Worked	2
3	Wage	$4.65
4		
5	Regular Pay	=B1*B3
6	Overtime	=B2*B3*1.5
7	Total	=B5+B6

Template 3

	A	B	C	D
1		Quantity	Price	Total
2	Small			=B2*C2
3	Medium			=B3*C3
4	Large			=B4*C4
5				
6			Total	=SUM(D2..D4)

Template 4

	A	B	C	D
1	Name	Hours	Wage	Pay
2	Tamara	14	$6.50	=B2*C2
3	Jennifer		$5.00	=B3*C3

Template 5

	A	B
1	Initial Amount	$5000.00
2	Rate	0.05
3	Years	3
4		
5	Final Value	=((1+B2)^B3)*B1

Template 6

	A	B
1	Width	
2	Length	=30-(2*B1)
3	Perimeter	=(2*B1)+B2
4		
5	Area	=B1*B2

Template 7

	A	B
1	No. of Hotdogs	
2	No. of Hamburgers	
3		
4	Volume	=(120*B1)+(200*B2)
5	Profit	=0.75*B1+1.5*B2

Exercise 1

Open a new spreadsheet. Put the label Name in cell A1 and the names Ron, Sally, Jeff, Tina, Paula, Chris in cells A2 to A7. In cell B1, put the label Test 1. The marks for the test are 78, 54, 96, 58, 67, 75. Place these marks in cells B2 to B7. In cell C1 place the label Test 2. The marks for this test are 89, 65, 78, 84, 59, 86. Your spreadsheet should look like this:

	A	B	C	D
1	Name	Test 1	Test 2	
2	Ron	78	89	
3	Sally	54	65	
4	Jeff	96	78	
5	Tina	58	84	
6	Paula	67	59	
7	Chris	75	86	

Place the label Average in cell D1. In cell D2, place the formula =AVERAGE(B2..C2) or an equivalent formula suitable to your spreadsheet program. (Note: The formulas shown here are for ClarisWorks. Your spreadsheet's formulas may be different.) In the cell D3 place the formula =AVERAGE(B3..C3). Continue for rows 4 to 7.

To calculate the average for the whole class, you need to find the average of column D. Place the label Class Average in cell C8. This may not fit in the column. We will change it later. In cell D8 place the formula =AVERAGE(D2..D7). Your spreadsheet should now look like this:

	A	B	C	D
1	Name	Test 1	Test 2	Average
2	Ron	78	89	83.5
3	Sally	54	65	59.5
4	Jeff	96	78	87
5	Tina	58	84	71
6	Paula	67	59	63
7	Chris	75	86	80.5
8			Class Avera	74.1

Exercise 1 (continued)

To fit the label **Class Average** into column C change the column width. Consult your manual or go to your on screen **Help** menu. Notice that numbers are aligned to the right side of the columns. Investigate how to centre these numbers. The class average is 74.0833333. When you are finished, your spreadsheet should look like this:

	A	B	C	D
1	Name	Test 1	Test 2	Average
2	Ron	78	89	83.5
3	Sally	54	65	59.5
4	Jeff	96	78	87
5	Tina	58	84	71
6	Paula	67	59	63
7	Chris	75	86	80.5
8			Class Average	74.1

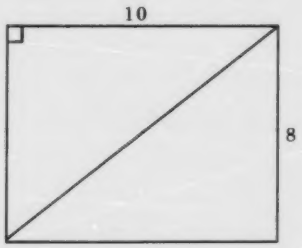
Questions

1. What is the difference between a row and a column?
2. What are the three types of information that can go into a cell? Explain.
3. Write the specific steps to (a) format a number to two decimal places, (b) format a number to show dollars and cents (currency), and (c) change column width.
4. What are the default settings for displaying numbers in your spreadsheet program?

Exercise 2

A template is a spreadsheet containing categories and formulas, but no data. The user can enter data wherever needed, and the quantitative values required will be calculated automatically by the formulas.

1. Given the rectangle below, it is easy to determine the perimeter, area, and diagonal length given the formulas below:



$P = 2(\text{Length} + \text{Width})$

$A = \text{Length} \cdot \text{Width}$, where P = Perimeter
D = Diagonal
A = Area

$D = \sqrt{\text{Length}^2 + \text{Width}^2}$

If you wanted to calculate rectangles with various lengths and widths, you would have to recalculate everything. Instead, you can use a template. Open Template 1.

	A	B	C	D
1	Length			
2	Width			
3				
4	Perimeter			
5	Area			
6	Diagonal Length			

Try entering information from the rectangle in the diagram. Enter the value 10 in cell B1 and 8 in cell B2. Notice how the spreadsheet program calculates the perimeter, area, and diagonal length. The formulas were placed in cells B4 to B6. Now move your cursor back to cell B1.

Questions

a) Use Template 1 to calculate the area, perimeter, and diagonal length of these rectangles.

Length	Width	Area	Perimeter	Diagonal
12	9			
18	5			
22	13			
23.5	17.32			

b) Move to cell B4. Write the formula in the cell. Do the same for cells B4 and B5. List the formulas for cells B4, B5, and B6 below.

B4 _____

B5 _____

B6 _____

Exercise 2 (continued)

2. Spreadsheets can be used to calculate payroll. Open Template 2.

	A	B
1	Reg Hours Worked	
2	OT Hours Worked	
3	Wage	
4		
5	Regular Pay	
6	Overtime	
7	Total	

a) Use Template 2 to calculate the data required to complete the table below:

Name	Reg Hours	OT Hours	Wage	Regular Pay	Overtime	Total
Bob	38	0	$7.85			
Ralph	40	5	$4.65			
Shelly	25	0	$8.00			
Diane	40	2	$7.50			
David	40	0	$7.42			

b) What are the formulas in the following cells in Template 2? List them below.

B5 _____

B6 _____

B7 _____

Exercise 2 (continued)

3. You are working in a hardware store that sells nails. Because the price often changes, you use the following template to calculate the total cost easily. Open **Template 3**. You should see the following:

	A	B	C	D
1		Quantity	Price	Total
2	Small			
3	Medium			
4	Large			
5			Total	

a) Use **Template 3** to calculate the data required to complete the table below:

No. of Sm	Cost Sm	Subttl	No.of Med	Cost Med	Subttl	No. of Lg	Cost Lg	Subttl	Total
25	$0.05		50	$0.07		10	$0.10		
30	$0.05		45	$0.07		20	$0.10		
40	$0.05		65	$0.07		50	$0.10		
50	$0.05		25	$0.07		40	$0.10		
75	$0.13		80	$0.15		60	$0.17		
100	$0.13		78	$0.15		80	$0.17		
45	$0.13		45	$0.15		70	$0.17		
50	$0.13		50	$0.15		85	$0.17		

b) What are the formulas in the cells below?

D2 _____

D3 _____

D4 _____

D5 _____

Exercise 3

The templates that you have used so far have been provided for you. Generally, for simple spreadsheets, it is not necessary to pay people to develop these spreadsheets. You can design them yourself. To do this, you must be able to write your own formulas.

Examples

	A
1	Quiz Mark
2	78
3	65
4	43
5	67
6	87
7	=SUM(A2..A6)
8	

	A	B
1	Budget	
2		
3	1994	$2000.00
4	1995	$2500.00
5	Difference	=B4-B3
6		
7		
8		

	A	B
1	Quantity	10
2	Price	$2.35
3	Subtotal	=B1*B2
4	Tax	=0.14*B3
5	Total	=B3+B4
6		
7		
8		

1. Create a new spreadsheet. Type in the labels as shown below and create your own formula to calculate the missing amount. When you are done creating the template save it as Prob1 and print it.

	A	B	C
1	Problem 1		
2	Your Name		
3		Hours Worked	35
4		Wage	$4.56
5		Pay	

2. Modify the spreadsheet of Problem 1 to look like the one below. Overtime pay is the wage multiplied by 1.5 multiplied by the number of hours over 40. Assume that the person may work over 40 hours. Create formulas for cells C5, C6, and C7 to calculate the missing amounts. Save your template as Prob2 and print it.

	A	B	C
1	Problem 2		
2	Your Name		
3		Hours Worked	42
4		Wage	$4.56
5		Reg Pay	
6		OT Pay	
7		Total	

Exercise 3 (continued)

3. Create a new spreadsheet to record a teacher's marks. The term mark is the average of the two tests. The final mark is 40% of the exam mark plus 60% of the term mark. Create formulas for calcuating a student's term and final marks. Enter these formulas into cells E2 to E7 and G2 to G7. Save the spreadsheet as Prob3 and print it.

	A	B	C	D	E	F	G
1	Problem 3	Name	Test 1	Test 2	Term Mark	Exam	Final
2	Your Name	Bob	70	75		65	
3		Susan	85	82		75	
4		Jeff	75	70		65	
5		Randi	96	89		82	
6		Carrie	56	65		51	
7		Scott	87	78		72	

Questions

a) Write a formula that you could use to add the values in cells C12 and C13.

b) Write a formula that you could use to find the difference between cells D10 and D9.

c) Write a formula that you could use to multiply cells A10, A11, and A12 together.

d) Write a formula that could be used to add cells B6 to B10.

Exercise 4

What if questions request a change of value to find (a) how it affects the overall picture or (b) what value you would need to achieve a certain result. Good examples are investments.

Suppose you need $10 000 in one year's time. You know that you can invest money at a 9.5% annual rate of interest. What initial amount do you need to invest to achieve this goal? The three spreadsheets below show the first steps in how you might use a series of trial investments to answer this problem. Continue these estimates until you can produce a value for B3 within one dollar of your target. Notice that 9.5% is expressed as 0.095.

	A	B
1	Initial Value	$8000.00
2	Rate	0.095
3	Final Value	$8760.00
4		

	A	B
1	Initial Value	$9000.00
2	Rate	0.095
3	Final Value	$9855.00
4		

	A	B
1	Initial Value	$9150.00
2	Rate	0.095
3	Final Value	$10019.00
4		

The formula in B3 is =B1+(B1*B2).

Generally, *What if* questions involve more complicated spreadsheets.

1. Open **Template 4**. You should see this spreadsheet.

	A	B	C	D
1	Name	Hours	Wage	Pay
2	Tamara	14	$6.50	$91.00
3	Jennifer		$5.00	$0.00
4				

Enter a number in cell B3 to determine how many complete hours Jennifer needs to work to make more than Tamara.

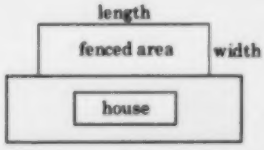
Exercise 4 (continued)

2. Open Template 5.

	A	B
1	Initial Amouont	$5000.00
2	Rate	0.05
3	Years	3
4		
5	Final Value	$5788.12
6		
7		

Roger has $5000. He would like to invest it to have $10 000 eventually. He thought that at 5% for three years, he would reach his goal. As you can see he was wrong. Beginning with $5000, find a combination of time and rate that will give him at least $10 000. Remember that 5% is expressed as 0.05. You are not allowed to use an interest rate greater than 15%. Find more than one combination.

3. Suppose you need to fence in an area next to your house. You have 30 m of fencing material and you want to fence in the largest possible area. What length and width should you use? A diagram might look like this:



Open Template 6. You should see this template:

	A	B
1	Width	
2	Length	0
3	Perimeter	0
4		
5	Area	0

Exercise 4 (continued)

What formulas are in these calculated cells?

B2 _____

B3 _____

B5 _____

Experiment substituting values for width into cell B1. What width creates the maximum area? Numbers may include decimals. Before continuing, learn how to save and print a spreadsheet without altering the template.

4. Open Template 7.

You are employed at a hamburger and hot-dog stand. Your employer asks you to take a mobile stand to a music festival. You want to make the most money possible (profit). Your employer instructs you that you must take at least 40 of each (hamburgers and hot dogs). You know that you can make \$1.50 profit for each hamburger sold and \$0.75 for each hot dog. The storage cooler on the mobile stand is limited to 20 000 cm^3. The volume of each hamburger is 200 cm^3. The volume of each hot dog is 120 cm^3. Experiment with the number of hamburgers and hot dogs that will maximize your profits within the storage limitations given. Write about your findings.

	A	B
1	No. of Hot Dogs	
2	No. of Hamburgers	
3		
4	Volume	
5	Profit	
6		

State the formulas in cells B4 and B5.

B4 _____

B5 _____

Questions

a) What is a template?

b) Write directions to print a spreadsheet without damaging a template.

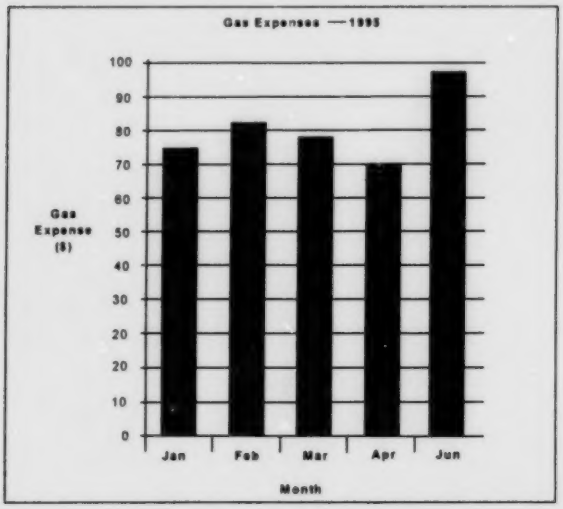
c) Explain in your own words how a spreadsheet can help someone solve a problem.

Exercise 5

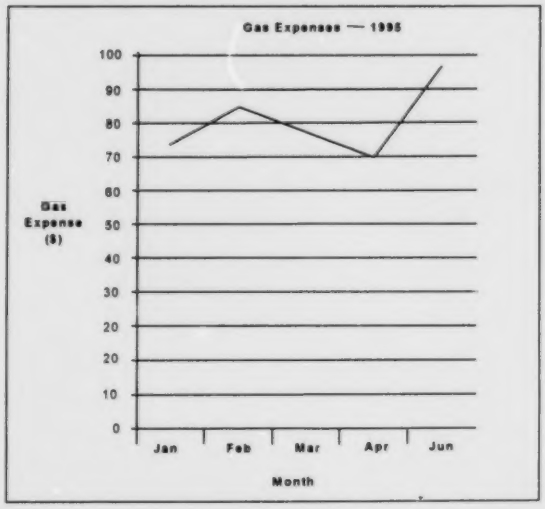
Most spreadsheets can also create graphs or charts. There are many types of graphs. You will be concerned with three: bar, line (or broken line), and pie.

Examples:

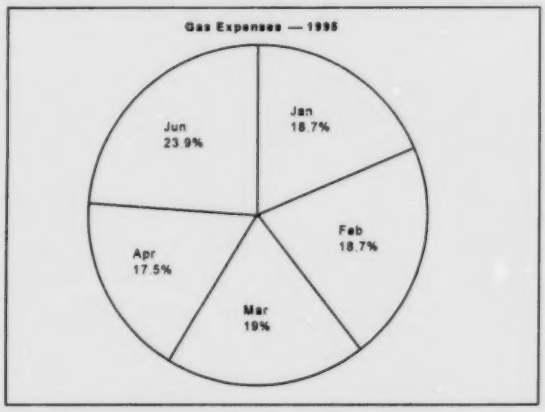
Bar Graph



Line Graph



Pie Graph



As you can see, each graph has its own characteristics. Some graphs are useful for some purposes and not for others. You must decide what type of graph to use for a particular purpose.

To create a graph:

1. Select data to be graphed (series).
2. Pick the appropriate type of graph.
3. Label the horizontal and vertical axes.
4. Display the graph.

Exercise 5 (continued)

Learn how to create these graphs using your spreadsheet program. Because printing graphs is sometimes time consuming, your teacher may ask you to display your graphs on-screen instead.

1. Create the spreadsheet shown below. When you are finished, create a bar graph showing the information. Show it to your teacher or print it.

	A	B
1	Name	Age
2	Bill	12
3	Roy	15
4	Sally	16
5	Karen	21
6	Donna	18

2. Create the spreadsheet below. When you are finished, create a broken line graph displaying the information. Show it to your teacher on-screen or print it.

	A	B
1	Date2	Temperature
2	Ap1	10
3	Ap2	11
4	Ap3	9
5	Ap4	7
6	Ap5	9
7	Ap6	12
8	Ap7	14

3. Create the spreadsheet below. When you are finished, display a pie graph of the information. Show it to your teacher on-screen or print it.

	A	B
1	Department	Budget Amount
2	Sales	$5000.00
3	Promotions	$2000.00
4	Rent	$1500.00
5	Utilities	$750.00
6	Insurance	$1350.00

Final Assignment

Make a spreadsheet using formulas that will do conversions between metric and imperial units. Use the following as a guide. You should be able to place any number in column A and get all the conversions for that value.

a) Create spreadsheets for length and area. Include metric and imperial units.

b) Then create spreadsheets for

- Volume: cm^3, m^3, $in.^3$, ft^3
- Time: seconds, minutes, hours, days, years

List the conversion formulas used.

Unit A
Spreadsheets
Solutions

Exercise 1 Solutions

1. A row is a horizontal string of cells. A column is a a vertical string of cells.
2. The three types of information are values, labels, and formulas.
3. Values can be changed, and formulas automatically reflect those changes.
4. Answers depend on spreadsheet used.

Exercise 2 Solutions

1. a)

Length	Width	Area	Perimeter	Diagonal
12	9	108	42	15
18	5	90	46	18.7
22	13	286	70	25.6
23.5	17.32	407.0	81.64	29.2

b) Answers depend on spreadsheet used.

2. a)

Name	Reg Hours	OT Hours	Wage	Regular Pay	Overtime	Total
Bob	38	0	$7.85	$298.30	$0.00	$298.30
Ralph	40	5	$4.65	$186.00	$34.88	$220.88
Shelly	25	0	$8.00	$200.00	$0.00	$200.00
Diane	40	2	$7.50	$300.00	$22.50	$322.50
David	40	0	$7.42	$296.80	$0.00	$296.80

b) Answers depend on spreadsheet used.

3. a)

No. of Sm	Cost Sm	Subttl	No. of Med	Cost Med	Subttl	No. of Lg	Cost Lg	Subttl	Total
25	$0.05	$1.25	50	$0.07	$3.50	10	$0.10	$1.00	$5.75
30	$0.05	$1.50	45	$0.07	$3.15	20	$0.10	$2.00	$6.65
40	$0.05	$2.00	65	$0.07	$4.55	50	$0.10	$5.00	$11.55
50	$0.05	$2.50	25	$0.07	$1.75	40	$0.10	$4.00	$8.25
75	$0.13	$9.75	80	$0.15	$12.00	60	$0.17	$10.20	$31.95
100	$0.13	$13.00	78	$0.15	$11.70	80	$0.17	$13.60	$38.30
45	$0.13	$5.85	45	$0.15	$6.75	70	$0.17	$11.90	$24.50
50	$0.13	$6.50	50	$0.15	$7.50	85	$0.17	$14.45	$28.45

b) Answers depend on spreadsheet used.

Exercise 3 Solutions

The formulas given as examples are ClarisWorks formulas.

1.

	A	B	C
1	Problem 1		
2	Your Name		
3		Hours Worked	35
4		Wage	$4.56
5		Pay	=C3*C4

2.

	A	B	C
1	Problem 2		
2	Your Name		
3		Hours Worked	42
4		Wage	$4.56
5		Reg Pay	=40*C4
6		OT Pay	=(C3-40)*C4*1.5
7		Total	=C5+C6

3.

	A	B	C	D	E	F	G
1	Problem 3	Name	Test 1	Test 2	Term Mark	Exam	Final
2	Your Name	Bob	70	75	=AVERAGE(C2..D2)	65	=(F2*0.4)+(E2*0.6)
3		Susan	85	82	=AVERAGE(C3..D3)	75	=(F3*0.4)+(E3*0.6)
4		Jeff	75	70	=AVERAGE(C4..D4)	65	=(F4*0.4)+(E4*0.6)
5		Randi	96	89	=AVERAGE(C5..D5)	82	=(F5*0.4)+(E5*0.6)
6		Carrie	56	65	=AVERAGE(C6..D6)	51	=(F6*0.4)+(E6*0.6)
7		Scott	87	78	=AVERAGE(C7..D7)	72	=(F7*0.4)+(E7*0.6)

Questions

Answers depend on spreadsheet used. ClarisWorks answers are as follows:

a) =C12+C13

b) =D10 – D9

c) =A10 * A11 * A12

d) =SUM(B6..B10)

Exercise 4 Solutions

What if questions were emphasized as a major advantage of spreadsheets. Your teacher may or may not wish to have you print your spreadsheets.

1. **(Template 4)**

   Jennifer needs to work 18.2 hours to make exactly $91.00.

2. **(Template 5)**

   There are a variety of combinations. Teachers may want to discuss the reason why students were not allowed to use a maximum rate. Teachers could also impose other limits, such as investing for fewer than 10 years.

3. **(Template 6)**

   A quadratic equation could be used to solve this problem.

   A width = 7.5 and a length = 15 give the greatest area.

   Answers depend on spreadsheet used.

4. **(Template 7)**

   This is an example of a linear programming exercise. At this level, it is advisable to have students solve this problem using trial and error. Teachers may want to modify the limits in the question.

   Answers depend on spreadsheet used.

   Questions

   a) A template is a spreadsheet that has formulas already in place and manipulates data as the data are entered. The user only needs to enter data.

   b) Answers depend on software used.

   c) Students should find two or three areas in which spreadsheets are used (e.g., service department of car dealership, estimator at kitchen cabinet retailer, farm fertilizer dealer).

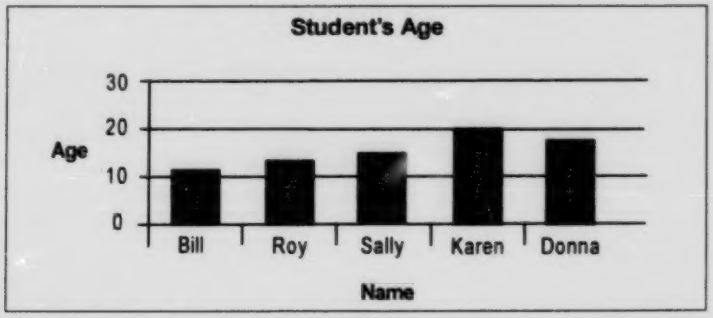
Exercise 5 Solutions

If you do not know how to make a graph using a spreadsheet, ask your teacher. The visual benefits of displaying numbers in a chart or graph outweigh the time required to create the chart or graph.

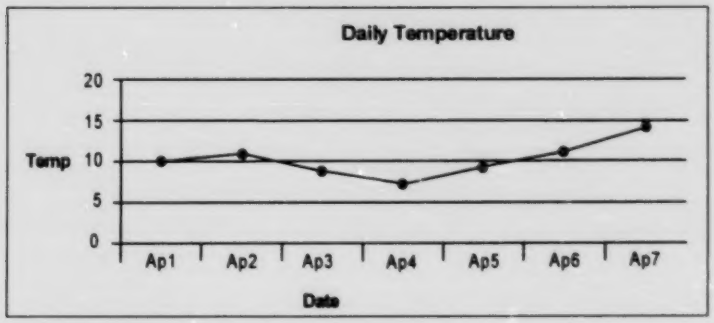
Graphs generally take a long time to print. Your teacher may want to evaluate your graphs from the screen display.

If graphs are used in the linear models unit, it is suggested that broken line graphs (line graphs) be used. Ensure that the x-axis appears on the graph so that the y-intercept can be read.

1.



2.



3.

Final Assignment Solutions

This final assignment may be used for evaluation. Save this spreadsheet as Convert and refer to it throughout the course when conversions are necessary. These charts will be helpful in the unit on Measurement Technology.

a) **Length**

	A	B	C	D	E	F	G	H	I
1	Number of		cm	m	km	inches	feet	yards	miles
2	1	cm	1	0.01	0.00001	0.3937	0.03281	0.01094	6.21e-6
3	1	m	100	1	0.001	39.97	3.281	1.094	6.21e-4
4	1	km	100000	1000	1	39370	3281	1094	0.6214
5	1	inches	2.54	2.54e-2	2.54e-5	1	0.0833	0.0278	1.57e-5
6	1	feet	30.48	3.04e-1	3.04e-4	12	1	0.333	1.89e-4
7	1	yards	91.44	9.14e-1	9.14e-4	36	3	1	5.68e-4
8	1	miles	160930	1609.3	1.6093	63360	5280	1760	1

Area

	A	B	C	D	E	F	G	H	I
1	Number of		cm^2	m^2	sq inches	sq feet	acre	sq mile	hectare
2	1	cm^2	1	0.0001	0.155	1.076e-8	2.47e-8	3.8e-11	1e-8
3	1	m^2	10000	1	1550	10.76	2.47e-4	3.86e-7	0.0001
4	1	sq inches	6.452	6.452e-4	1	6.944e-3	1.59e-7	2.4e-10	6.45e-8
5	1	sq feet	929	0.0929	144	1	2.29e-5	3.58e-8	9.29e-6
6	1	acre	4.047e+7	4047	6.273e+6	43560	1	1.56e-3	0.4047
7	1	sq mile	2.59e+10	2.590e+6	4.007e+9	2.788e+7	640	1	259
8	1	hectare	1.000e+8	1.000e+4	1.550e+7	1.976e+5	2.471	3.86e-3	1

b) **Time**

	s	min	h	d	y
1 second	1	0.01667	2.778×10^{-4}	1.157×10^{-5}	3.169×10^{-8}
1 minute	60	1	0.01667	6.994×10^{-4}	1.901×10^{-6}
1 hour	3600	60	1	0.04167	1.141×10^{-4}
1 day	8.640×10^{4}	1440	24	1	2.738×10^{-3}
1 year	3.156×10^{7}	5.259×10^{5}	8766	365.3	1

Final Assignment Solutions (continued)

Volume (Capacity)

	cm^3	m^3	in^3	ft^3
1 cubic centimetre	1	10^{-6}	0.06102	3.531×10^{-5}
1 cubic metre	10^6	1	6.102×10^4	35.31
1 cubic inch	16.39	1.649×10^4	1	5.787×10^{-4}
1 cubic foot	8.832×10^4	0.02832	1728	1

Unit B
Exploring Mathematics Using Technology

These exercises can be interspersed throughout the year depending on the content that is being introduced.

Exercise 1

1. The number systems studied are rational, irrational, whole, and integer. State all the number systems to which the following numbers belong.

   a) $-\frac{3}{8}$

   b) 14

   c) 2.3233233323333 ...

   d) 0

   e) 4.1

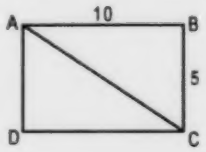
   f) $\sqrt{7}$

   g) -12

   h) $7\frac{3}{4}$

   i)

   Area of given circle.

   j) 

   Length of AC

2. a) Describe in writing the distinction between rational and irrational numbers.

   b) Describe in writing the distinction between a whole number and a rational number. Can any numbers belong to both number systems? Explain.

3. Write the value of each as a number rounded to two decimal places.

   a) π

   b) 17.5962

   c) 15.4449

   d) $\sqrt{2}$

   e) $\sqrt{3}$

   f) $\sqrt{2} \times \sqrt{3}$

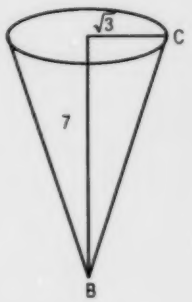
   Compare the answer of (f) with the product of the answers of (d) and (e).

4. For the given cone,

   a) use your calculator to find

   - the length of BC correct to three decimal places
   - the diameter of the base of the cone correct to two decimal places

   b) state the length of BC in simplified radical form



Exercise 1 (continued)

5. Using a scientific/graphing calculator, find the answers to each of the following. (State all answers in scientific notation with five or fewer digits and record the keying sequence.)

a) $\dfrac{64.2\times 386}{4.09\times 527}$

b) $\dfrac{-11.011+0.953}{1.72\times 0.3418}$

c) $(42.7)^3$

d) $(4.23\times 79.3)^{\frac{-5}{4}}$

e) $\sqrt[4]{176.82}$

f) $\dfrac{0.0469+0.0371}{1.28+8.21}$

g) $\cos 3.2$

h) $\sin 27.3^0$

i) $\dfrac{5.7^2-3.6^2-4.2^2}{(-2)(3.6)(4.2)}$

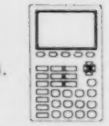
j) $\left[\dfrac{1.8265\times\sqrt[3]{50.293}}{(1.397)^4\times\sqrt{68.465}}\right]$

6. a) Find the area of a triangle by using the formula $A=\frac{1}{2}bh$ with base b of $12\frac{1}{4}$ in. and height h of $7\frac{1}{8}$ in. correct to two decimal places.

   b) Find the cost of carpeting a rectangular room with length 7.6 m and width 5.1 m if the cost of carpet is \$20.79/m^2.

   c) If you were to solve for r in the following equation, $6000 = 1526\ (1 + r)^4$, you would get $r=\sqrt[4]{\dfrac{6000}{1526}}-1$. Find r correct to three decimal places.

Exercise 2



Use a graphing calculator or computer graphing software to answer the following questions.

1. Graph the following equations:

   a) $y = 4x - 6$ b) $2x - 5y = -6$

   c) $y = 1.5x + 8$ d) $4x + 3y + 2 = 0$

   What do you see as similarities and differences in the graphs?

2. Graph $y = 3x + 5$.

   a) What is the effect of changing the value of 5?

   b) What is the effect of changing the value of 3?
   (**Note**: Changes could include negative values.)

3. Graph the following using the zoom features where necessary to get a good view of the graph. At what values of x does each graph cross the x-axis?

   a) $y = 3x^2 - 1$ b) $y = -2x^2 + 3$

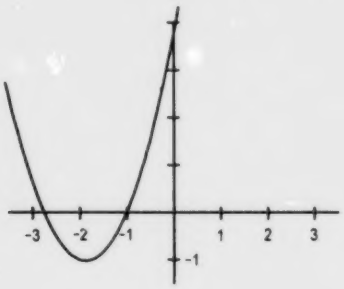
   c) $y = 2x^2 - x - 3$ d) $y = -4x^2 - 3x + 1$

4. Investigate how the graph of $y = ax^2$ changes as the value of a changes. Include negative values of a.

5. Graph the equation $y = 2x^2 \times 3x + 4$. This equation is written in the form $y = ax^2 + bx + c$, and the value of c is 4. Explain what happens to the graph as c changes.

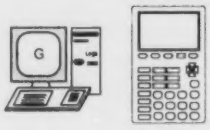
6. Graph the equation $y = x^2 + 3x + c$. This equation is also written in the form $y = ax^2 + bx + c$, and the value of b is 3. Explain what happens to the graph of $y = ax^2 + bx + c$ as b changes.

7. Exploration question: Examine the given graph. Write an equation in the form $y = ax^2 + bx + c$ that you think would produce this graph. Use your graphing calculator to explore and improve your guesses.



Exercise 3

Use a graphing calculator or computer graphing software to answer the following questions.



1. Graph each of the following. Verify the domain and range as appropriate. Describe the shape of each of the graphs.

   a) $y = \sin x$ (x in rads or degrees)
   b) $y = \cos x$ (x in rads or degrees)
   c) $y = 3^x$
   d) $y = 0.75^x$
   e) $y = |x| - 3$
   f) $y = |2x - 5|$

2. By tracing the function and verifying the domain and range as required, approximate the zeros (if applicable) for each of the following equations. (Real zeros are located where the graph hits the x-axis, that is, where $y = 0$.)

   a) $y = 1.5x + 8$
   b) $y = 2x^2 - 5x - 4$
   c) $y = x^3 + x^2 - 3x - 1$
   d) $y = 2^x$
   e) $y = 3(\sin x) + 1$

3. For the following equations, find any places where there are no values of y for a value of x.

   a) $y = \dfrac{2}{x - 3}$
   b) $y = \sqrt{9 - x^2}$

4. You are in charge of a charitable organization where your profit or loss (y) is dependent on the number of raffles you conduct (x) during a given year. You find that the situation you are in fits the equation $y = 4x - 28$.

   Graph the results and find where $y = 0$. This will indicate how many raffles you must conduct to break even. (Hint: Use the trace feature.)

5. In a specific case, the trajectory of a baseball after it leaves a bat can be described by the equation $y = -0.05x^2 + 5.4x$ where y is the height of the ball when it has travelled x yards from home plate.

   a) Graph the equation.
   b) Using zoom and trace, find the maximum height reached by the ball.
   c) Find the horizontal distance of the ball from home plate when it reached its greatest height.
   d) Find the total horizontal distance travelled by the ball (x-axis can be considered the ground).

Unit B
Exploring Mathematics Using Technology
Solutions

Exercise 1 Solutions

1. a) rational, real
   b) whole, integer, rational
   c) irrational
   d) whole, integer, rational
   e) rational
   f) irrational
   g) integer, rational
   h) rational
   i) with r = 14, the area is irrational
   j) irrational because $\sqrt{10^2+5^2} = \sqrt{125}$ does not appear to terminate or recur

2. a) All rational numbers can be named as functions.

   For example: $0.206 = \frac{206}{1000}$

   Some rational numbers become recurring decimals.

   For example: $\frac{7}{11} = 0.636363\ldots$, $\frac{2}{7} = 0.\overline{285714}$

   All recurring decimals can be renamed using fractions.

3. a) 3.14 b) 17.60
   c) 15.44 d) 1.41
   e) 1.73 f) 2.45

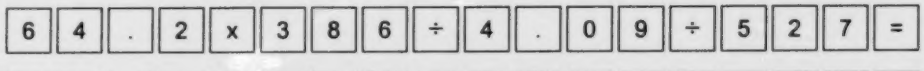
   The product of (d) and (e) is 2.4393 which, when rounded, equals 2.44. The value for (f) is more accurate. Calculator solutions should be rounded only in the last step.

4. a) Length of BC = 6.782
   Diameter of base = 3.46
   b) $\sqrt{49+3} = \sqrt{52} = \sqrt{4\times 13} = 2\sqrt{13}$

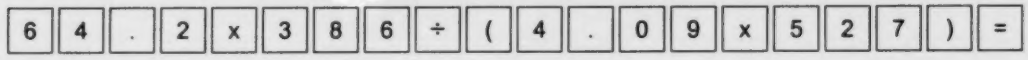
Exercise 1 Solutions (continued)

5. a) $11.4971 = 1.1497 \times 10^{1}$

among others

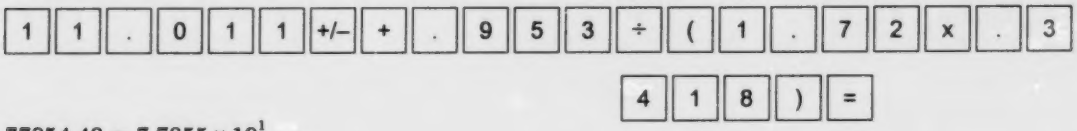


or

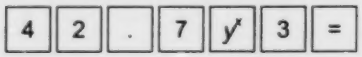


b) $-30.59371 \approx -30.594 = -3.0594 \times 10^{1}$

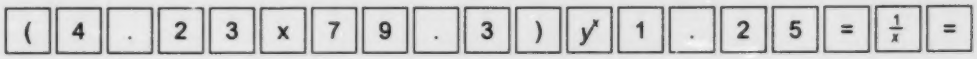
among others



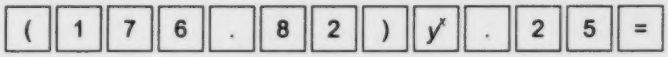
c) $77854.48 \approx 7.7855 \times 10^{1}$



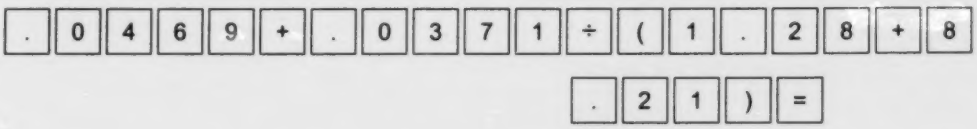
d) $.000039172 \approx 3.9172 \times 10^{-5}$



e) $3.646555 \approx 3.6466$



f) $.0088514^{3} \approx 8.8514 \times 10^{-3}$



g) Assuming radians, $-0.9982947 \approx -9.9829 \times 10^{-1}$, if degrees are assumed, $0.9984407 \approx 9.9844 \times 10^{-1}$

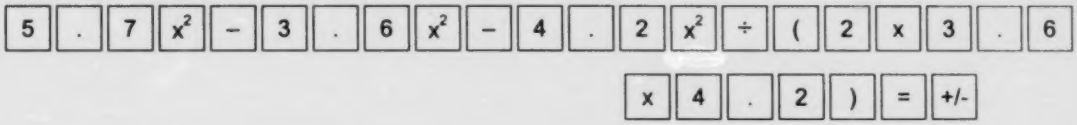
— calculators differ

h) $0.4586495 \approx 4.5865 \times 10^{-1}$, if degrees are assumed, $0.9984407 \approx 9.9844 \times 10^{-1}$

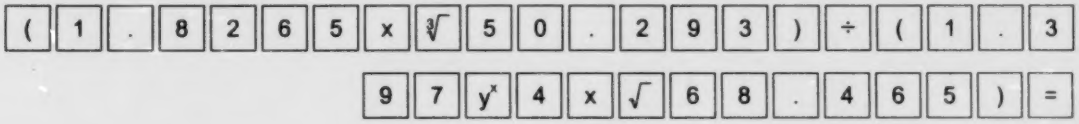
— calculators differ

Exercise 1 Solutions (continued)

i) $1.89 \div 30.24 = .0625 = 6.25 \times 10^{-2}$



j) $2.139286\ 2.1393 \times 10^{-1}$



— some calculators differ

6. a) 43.64 square units
   b) $805.82
   c) 0.408

Exercise 2 Solutions

1. Explore slopes and intercepts.

2. a) It shifts the curves (and intercept) up or down.
   b) It changes the slope.

3. a) 0.57735 and –0.57735
   b) 1.22474 and –1.22474
   c) 1.5 and –1
   d) –1 and $\frac{1}{4}$

4. For positive values of a, the parabola opens upward. For negative values of a, the parabola opens downward. The larger the absolute value of a, the wider the parabola.

5. The graph moves up or down.

6. The curve moves left or right.

7. Answers will vary.

Exercise 3 Solutions

1. Descriptors will vary.

2. a) –0.1875
   b) 3.13746 and –0.63746
   c) Approximately –2.2, –0.3, and 1.5
   d) There is none.
   e) 3.5 radians

3. a) At $x = 3$
   b) At $x > 9$ and $x < -9$

4. When $y = 0$, $x = 7$.
   Therefore, the number of raffles needed to break even is 7.

5. b) Maximum height = 145.7 yards
   c) Horizontal distance at maximum height = 53.4 yards
   d) Total horizontal distance = 108 yards

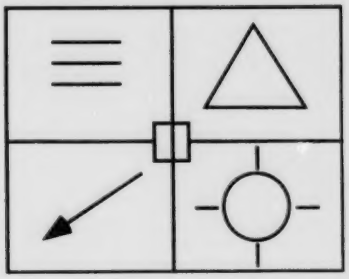
Unit C
Technical Communication

Exercise 1

Activity

Organization: Groups of 3 or more

Materials: — the following geometric configuration on a 4-by-5 card.



— paper and pencils

Directions:

1. One person describes the given geometric configuration using mathematical language as much as possible. No hand movements are allowed.
2. Without seeing the 4-by-5 card, the others in the group try to draw what is described.

Exercise 2

Students design a creature with a certain number of geometric shapes. They write a list of instructions for making this creature and then test the instructions on someone else by having him/her draw the creature based on the instructions. The second person critiques the instructions by comparing his/her drawing to the original drawing. The original writer than revises his/her instructions for drawing the creature.

Example (including assessment)

1. Student A draws a creature robot using each of the following shapes at least once: equilateral triangle, right-angle triangle, rhombus, rectangle, kite, square, and trapezoid.
2. Student A writes the instructions for someone else to draw the robot.
3. Student A gives the instructions to student B who draws the robot.
4. Students compare the drawing in number 3 to the original drawing. (The teacher assesses this drawing out of 10.)
5. Student B assesses the instructions out of 10 and meets with the writer to revise the instructions.
6. A third person, Student C, now follows the revised instructions to draw the robot. Student C's drawing is compared to the original by the teacher and marked out of 10. The revised instructions are also marked by Student C out of 10.
7. Total possible marks for this project is 50. This project takes about three to four classes to complete and assess.

Exercise 3

The following technical communication activities can be interspersed throughout the year.

1. Write directions for different "trips." Some examples are: from home to school, from school to home, from school to the location of the next inter-school sports event, from school to the band festival, from school to the local movie theatre, and from one student's house to another student's house.
2. Write a set of instructions to program a VCR to record four different shows over two weeks. This problem could start with writing the instructions for recording one show and then for recording multiple shows.
3. Follow the instructions in a calculator manual to perform a specific operation or operations. The calculator may be scientific or graphing.
4. Describe the procedure for performing a specific skill, such as shovelling a walk, mowing the grass, or playing a simple tune on a piano, drums, or flute. The procedure could also be a description of a particular athletic skill such as hitting a golf ball, throwing a football, hitting a baseball, shooting a wrist shot in hockey, shooting a foul shot in basketball, serving a volleyball, and so on.
5. Write instructions for someone else to be able to read the scale on a caliper or micrometer or other precision measurement device.
6. Describe the procedure for purchasing gas from a self-serve station and also from a full-service station.
7. Describe the procedure of checking oil in a vehicle.
8. Describe any home task such as making a pot of coffee, setting up a bread maker to have bread ready the next morning, preparing a complete meal, or making a dessert.
9. Follow the instructions in a computer software manual to make charts from data that have been captured in a spreadsheet.

Exercise 4

Pick up a discount advertisement from the newspaper. Have one class member describe the advertisement and the value of the discount exactly as shown. Then ask the listeners the following:

a) What was the regular price?

b) What was the rate of discount?

c) When is the expiration date?

d) If the regular price had no discount, what would be the total cost of the item including sales tax?

e) Make up an advertisement for the same item and same discount but with communication improvements.

Unit C
Technical Communication Solutions

Exercises 1, 2, 3, and 4 Solutions

Answers will vary.

Examples of student instructions and drawings are provided below for Exercise 2.

Exercise 2 — Example 1 of Student Instructions

Instructions for Bleepo

Measurements do not matter.

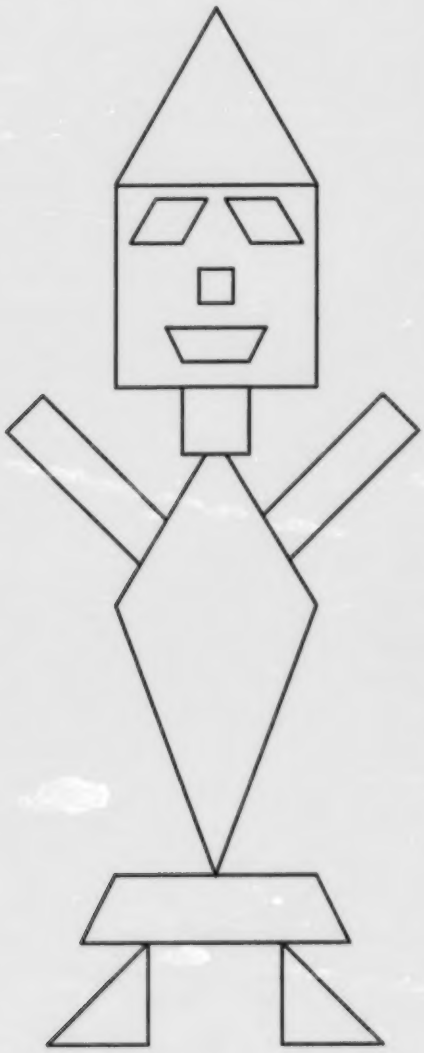
1. Draw a square three-quarters up the page. This is the head.
2. Draw an equilateral triangle on top of the square with the pointed part up. This will be the hat.
3. Underneath the head, draw a smaller square for his neck.
4. The body will be a kite. With the shorter sides pointing up, place the kite underneath the neck.
5. For the arms, use two rectangles. The first one should stick diagonally out from the shorter side of the body. The shorter side of the rectangle should be against the kite. Do the same with the second arm except on the other side of the body.
6. Bleepo wears a kilt. The kilt is a trapezoid. With the shorter side facing up, put it underneath the pointed part of the body.
7. Oh no! Bleepo needs some legs. They will be right-angled triangles. With the hypotenuse facing diagonally left (up), put one triangle underneath the trapezoid. Do the same with the other triangle except the hypotenuse should be facing diagonally right (up).
8. Make two rhombuses with the top points facing each other for the eyes.
9. Make a tiny little square for the nose.
10. Bleepo needs a mouth! Draw one for him. Draw a smaller trapezoid than the kilt for the mouth.

Revisions

5. Use two rectangles for the arms. Place the width side of one rectangle against the top part of the body, facing up. Do the same with the second rectangle except on the other side.
7. Use two right-angled triangles for the legs. With the angles on the bottom facing each other, put them underneath the trapezoid.

Exercise 2

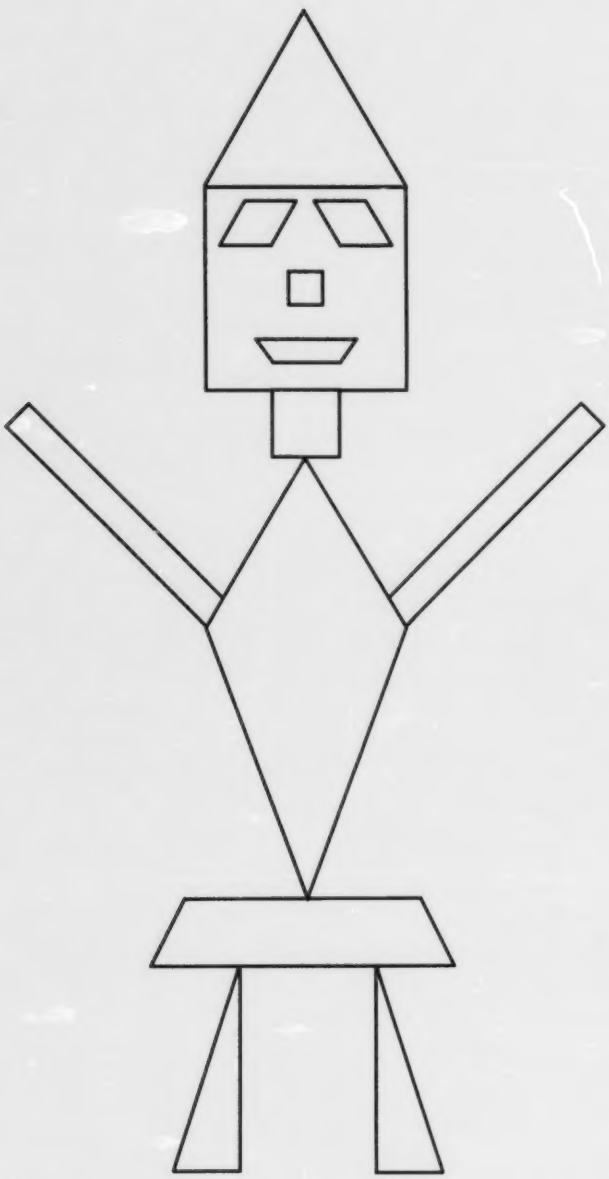
Example 1 — Original Drawing



"Bleepo"

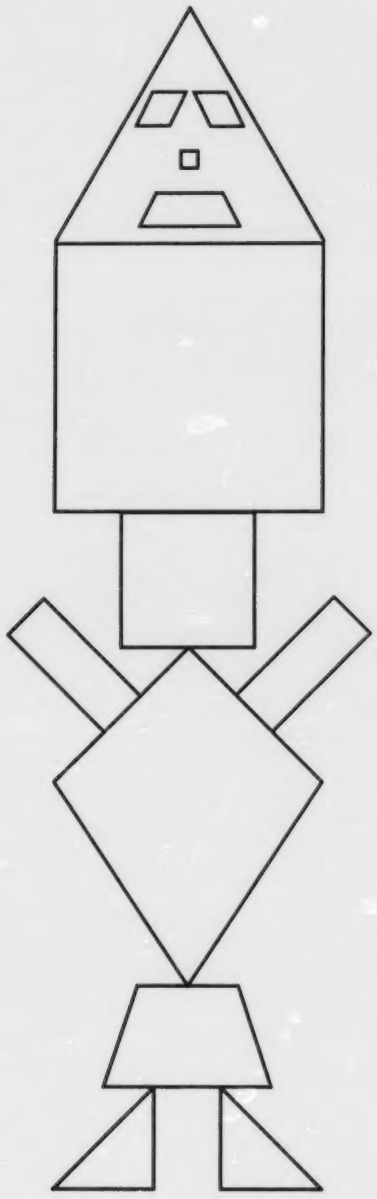
Exercise 2

Example 1 — First Drawing



Exercise 2

Example 1 — Second Drawing



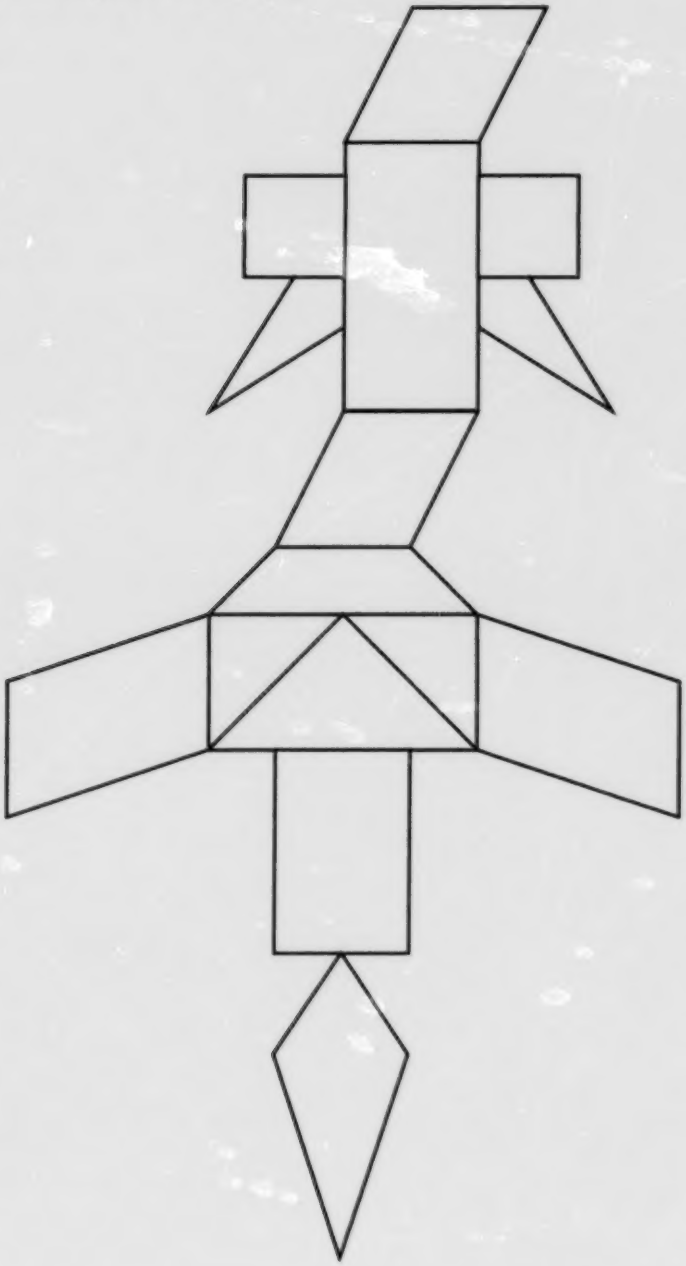
Exercise 2 — Example 2 of Student Instructions

Instructions

1. Position the paper as this one is.
2. Place a rhombus at the top with the top of the slant pointing right.
3. Put a rectangle directly underneath the rhombus, so that the top of the rectangle is in direct contact with the bottom side of the rhombus. Measurements are 5 cm long and 2.6 cm wide.
4. Place two squares on either side of the rectangle, 5 mm down from the top of the rectangle. Squares have equal sides, 2.1 cm.
5. Directly underneath the squares put two kites going out diagonally from the corner of the square and the sides of the rectangle. The longer bottom side of the kite is 2.5 cm long, the short side is 1.1 cm long.
6. Insert the next rhombus directly underneath the rectangle so the bottom of the slant is pointing left. The measurements of the rhombus are 2.5 cm per side.
7. Directly underneath the rhombus, place a trapezoid, length at top 2.5 cm, with each side slanting opposite. The left side slants to the left and the right side comes out to the right. The sides are 2.5 cm long. The length of the bottom is 5.6 cm.
8. Directly beneath the trapezoid, place two right triangles and one equilateral triangle. Three vertices of the triangles meet in the middle of the base of the trapezoid. On each side of the equilateral triangle place two right triangles. Each triangle is 2.8 cm in width. For each of the right triangles, the length of the triangle is 3.1 cm. The bottom length of the equilateral triangle is congruent to the bottom length of the trapezoid.
9. Place two rhombi on either side of both of the right triangles. The rhombus points downward and the slanted side is 3.9 cm long. The vertical side is 3.2 cm long. The directions are the same for the other rhombus.
10. Directly under the equilateral triangle place a rectangle. The two short sides are horizontal, and the two long sides are vertical. The vertical sides are 4.5 cm long and the horizontal sides are 2.1 cm long.
11. Directly below the rectangle, place a kite. The overall length of the kite is 5.9 cm.

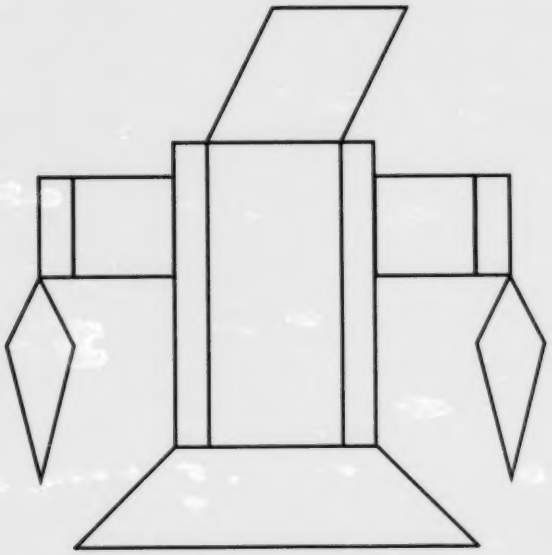
Exercise 2

Example 2 — Original Drawing



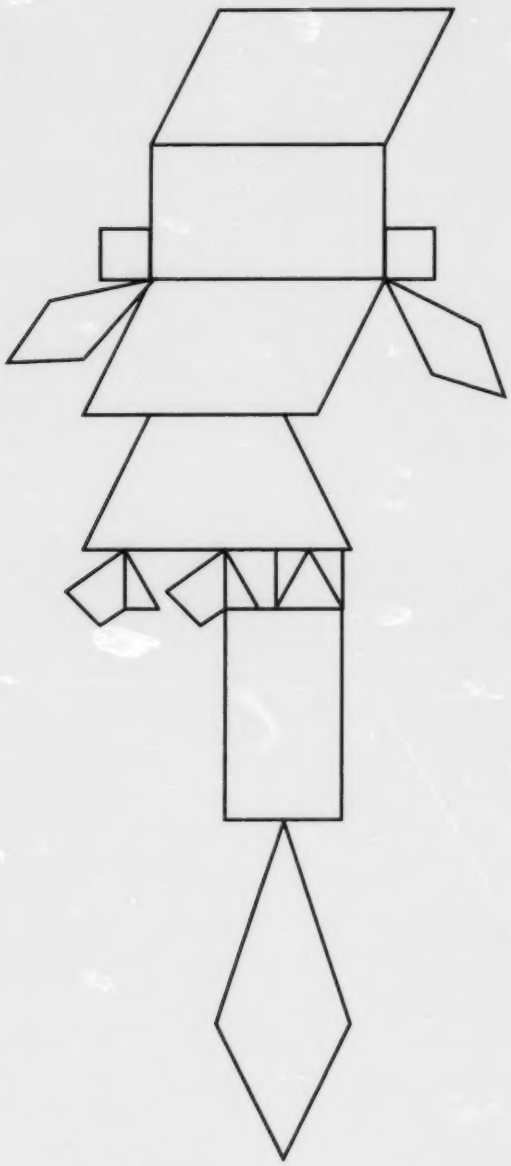
Exercise 2

Example 2 — First Drawing



Exercise 2

Example 2 — Second Drawing



Unit E
2D/3D Projects

Exercises for 2D and 3D Projects

The following formulas are to be used in these exercises. Use the value of π given by the calculator. Do not round until the final answer.

Area circle = πr^2

Area rectangle = lw

Area triangle = $\frac{bh}{2}$

Surface area sphere = $4\pi r^2$

Surface area cylinder = $2\pi r^2 + 2\pi rh$ *(with ends)*

Surface area cube = $6s^2$ (s = length of side)

Surface area cone = πrs (s = slant height)

Volume sphere = $\frac{4}{3}\pi r^3$

Volume cylinder = $\pi r^2 h$

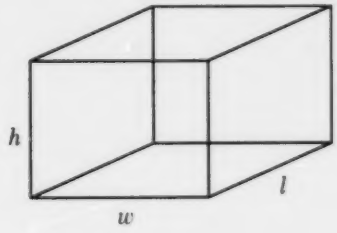
Volume cube = lwh

Volume pyramid = $\frac{1}{3}$ (area base) x height

Volume prism = (area base) x height

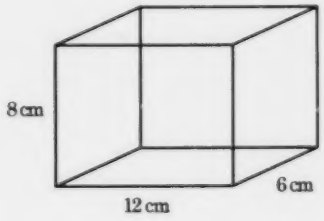
Exercise 1

1. The volume of this rectangular solid is the area of the base x height.

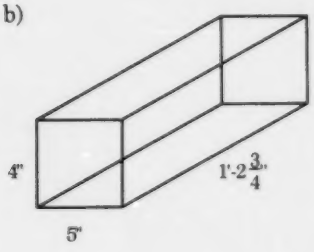


Find the volume of these figures:

a)



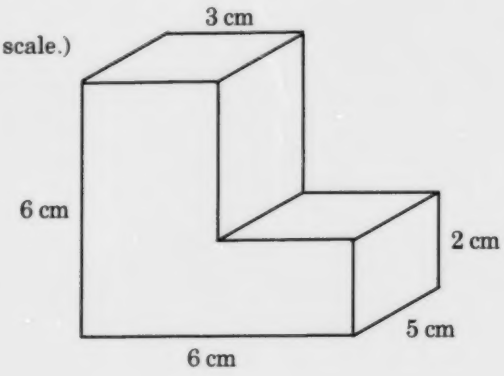
b)



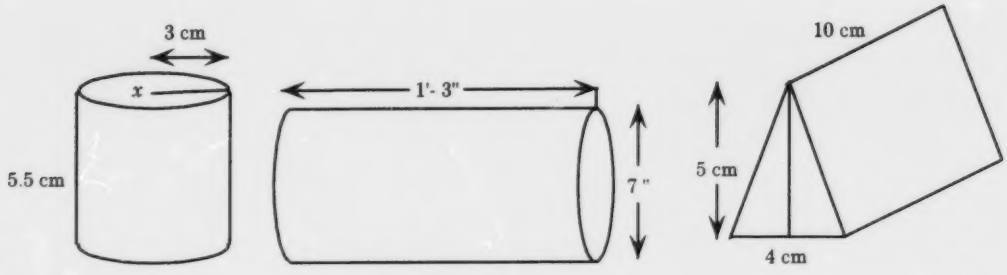
Exercise 1 (continued)

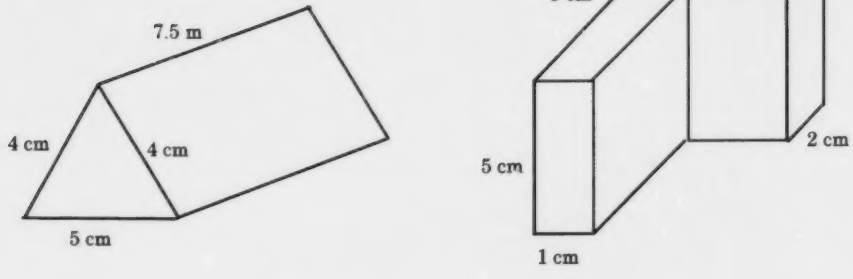
2. There are numerous methods to find the volume of this figure. Find its volume in two different ways. For each method you use, write directions for finding the volume.

(Diagram is not to scale.)



3. Find the volume of each of the following prisms.

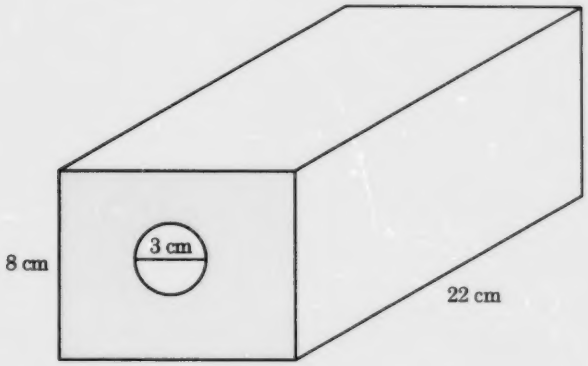




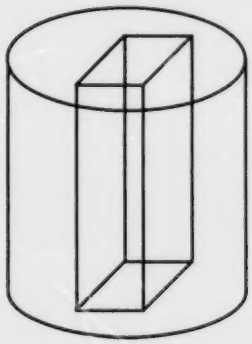
Exercise 1 (continued)

4.. Find the volume of these figures with holes in them.

a)

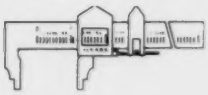


b)



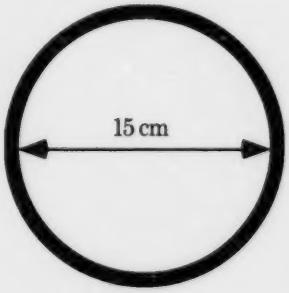
In this figure, the rectangular hole is $1\frac{1}{4}$" by $\frac{3}{4}$". There is $\frac{1}{2}$" from the hole to the edge on the left and right and $\frac{1}{4}$" from the hole to the edge at the front and back.

5. Since 1 cm^3 = 1 mL, how many times can you brush your teeth with 1 tube of toothpaste, assuming 1 cm of length per brush?



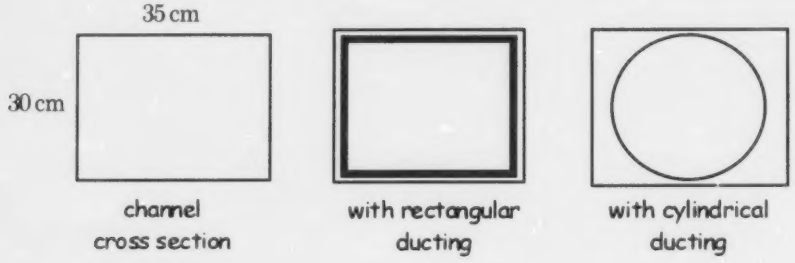
Hints: Find the number of millilitres of toothpaste in the tube. Measure the diameter of the hole at the end of the tube. Calculate the volume of 1 cm of length of toothpaste.

6. In the diagram you are shown a cross section of a 8.5 m length of pipe. If the rate of flow is 500 mL/s, how long will it take for the pipe to fill a 10 L tank?



Exercise 1 (continued)

7. A homeowner is installing new ducts for a heating system in the basement. Ducting for the 35-cm by 30-cm channel between joists can be manufactured in two cross section shapes: rectangular and cylindrical.



   a) Calculate the surface area and the capacity of a 5.5 m section length of each type of duct.

   b) What is the cost of metal ducting for each type if metal costs \$1.20/m^2?

   c) Which duct costs less per cubic centimetre of capacity?

8. The inside diameter of a copper pipe leading to a hot water faucet from a hot water tank in the basement is 1.75 cm. It is determined that water flows at 20 mL/sec and that it takes 55 seconds for hot water to arrive. Calculate the length of the hot water pipe.

9. A cord of wood is a pile of wood occupying a space 4' x 4' by 8'. A cord of wood is ordered and delivered in a truck whose cargo box is 4'-6" x 7'-3". How high must the wood be piled into the truck to load a cord of wood?

10. A board foot is a unit of volume that equals 144 cubic inches of lumber. Lumber yards price lumber per board foot.

    You are building a fence. You need four 4" x 4" posts (3.5' long), twenty-four 1" x 4" (3' long), and six 2" x 4" boards (4' long). Using the price of \$2.25/board foot, calculate the total cost including PST and GST.

Exercise 2

1. Calculate the volume of the following spheres where r = the radius, and d = diameter.

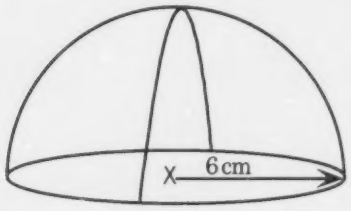
   a) $r = 5$ cm b) $r = 7\frac{1}{2}$" c) $d = 11.25$ cm

2. Calculate the surface areas of the following spheres:

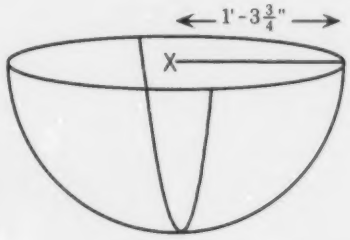
   a) $r = 3.34$ cm b) $r = 1'\text{-}2\frac{1}{2}$" c) $d = 14.05$ cm

3. Calculate the volumes and surface areas of the following hemispheres:

   a)



   b)



4. Assume that the radius of Earth is 6.37×10^6 m.

   a) Find the surface area of Earth in (i) square miles (ii) km^2.

   b) Find the volume of Earth in (i) cubic miles (ii) km^3.

5. Predict what will happen to the surface area and volume of a sphere when the radius is doubled. Verify it by calculating the surface area and volume of spheres with radii of 2 cm and 4 cm.

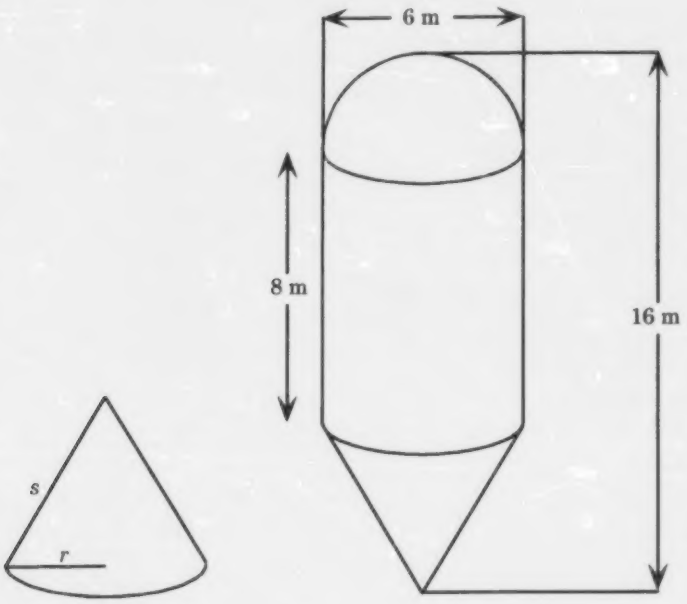
6. A balloon has a spherical shape and a diameter of 4 m. If its volume is increased by 30 m^3, what will be the new values for the diameter, volume, and surface area?

7. A spherical balloon is inflated at a constant rate. It takes 5 seconds for the radius to become 5". How many more seconds will it take for the radius to become 10"?

Exercise 2 (continued)

8. A grain storage bin has a cone-shaped lower end, a hemispherical top, and a cylindrical centre. One litre of paint covers 2.5 m^2 and costs \$9.50. Calculate the cost of the paint needed to paint the bin with one coat of paint.

   Recall: The formula for the surface area of a cone is SA = πrs where s is the slant height.



Exercise 3

1. a) Calculate the area of each circle (to two decimal places):

   i) $r = 3$ cm ii) $r = 6$ cm iii) $r = 9$ cm

   b) What happens to the area of a circle when you double the radius? What happens when you triple it?

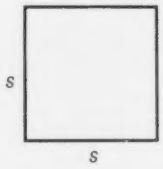
   c) What happens to the area of a circle when you multiply the radius by a factor of x?

   d) Use the area of a circle with a radius of 3 cm and your rule from part (c) to predict the areas of the following circles:

   i) $r = 12$ cm ii) $r = 21$ cm iii) $r = 4$ cm iv) $r = 10$ cm

2. a) Calculate the area of the following squares with side s:

   i) $s = 3$ cm ii) $s = 6$ cm iii) $s = 9$ cm



   b) What happens to the area when you double the side? What happens when you triple it?

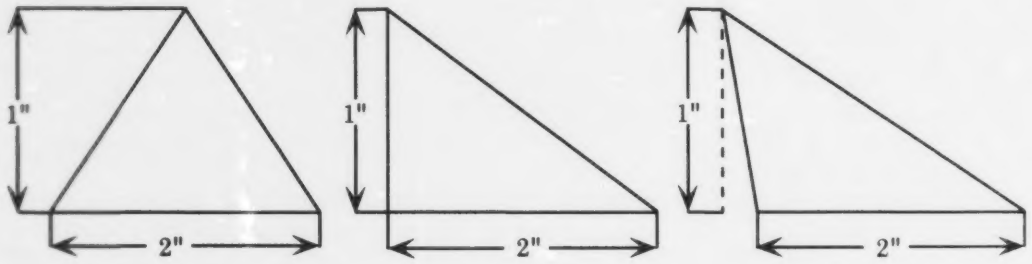
   c) What is the rule for the area of a square when you multiply the sides by a factor of x?

   d) Use the area of a square with a side of 3 cm and your rule from part (c) to predict the areas of the following squares:

   i) $s = 12$ cm ii) $s = 21$ cm iii) $s = 4$ cm iv) $s = 10$ cm

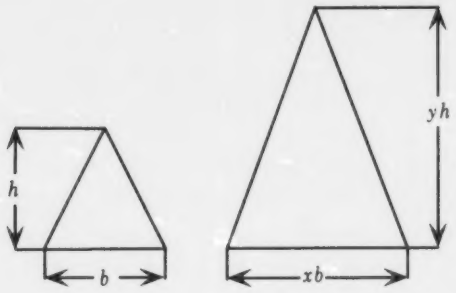
3. a) What is the rule that determines the area of a triangle when the base and height are both multiplied by the same factor x?

   b) Calculate the area of the following triangles. What can you determine from these calculations?



Exercise 3 (continued)

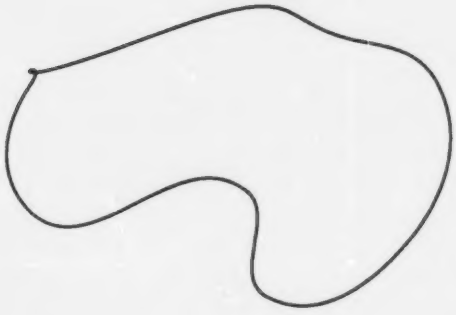
c) What is the rule for a triangle when the base is multiplied by a factor of x and the height by a factor of y?



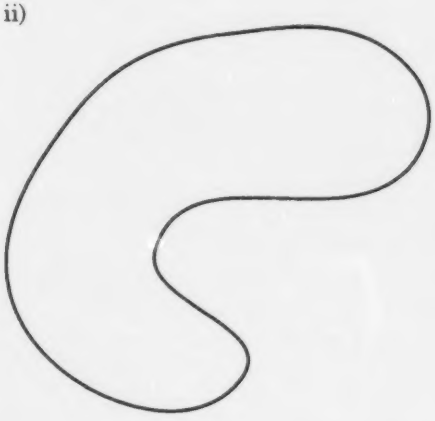
$A = \frac{1}{2}bh$ or $\frac{bh}{2}$ $\qquad$ $A = ?$

4. a) Use a plastic grid overlay to estimate the area of these figures. Use the rule that if the blob covers more than a half a square on your plastic overlay, you count the whole square. If it does not, you do not count it. How would you make a better estimate? Try your idea.

i)



ii)

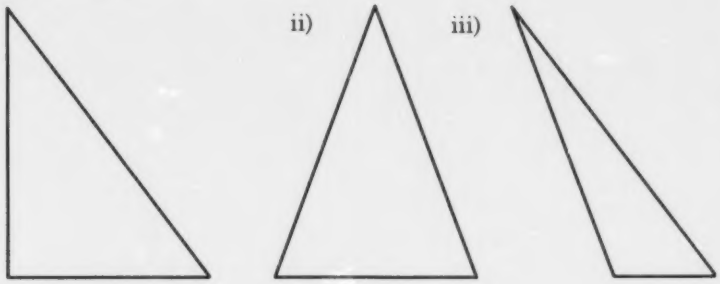


Teacher Note: *At the end of the exercises for this unit, two grids are provided that can be made into plastic grid overlays (see page E-23).*

Exercise 3 (continued)

b) Use the same plastic grid overlay to estimate the area of these triangles. Use the same rule for counting part squares.

i) ii) iii)



c) Heron's formula for the area of a triangle is

$$A = \sqrt{S(S-a)(S-b)(S-c)}, \text{ where } S = \frac{a+b+c}{2} \text{ (semiperimeter), and}$$

where a, b, and c are sides of the triangle.

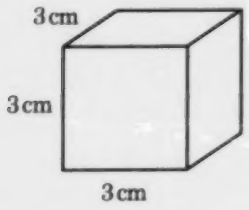
Using a ruler, measure the sides of the above triangles to the nearest millimetre. Use Heron's formula to find their areas and then compare these values to your estimates from part (b).

d) Estimate the areas of the triangles if all three sides are doubled. Check your estimates using Heron's formula.

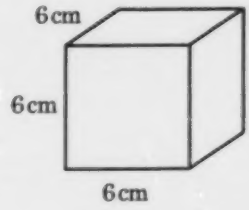
Exercise 3 (continued)

5. a) Find the surface area of each cube:

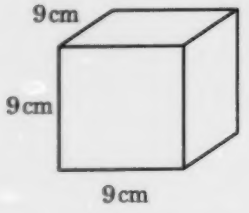
i)



ii)

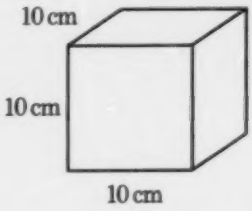


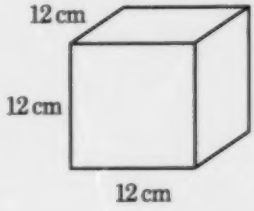
iii)



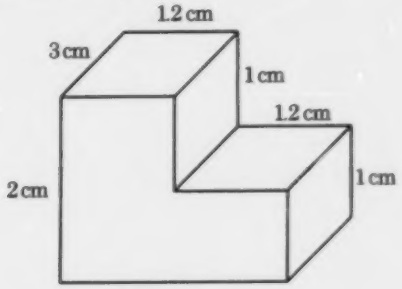
b) What is the rule for surface area where the sides of a cube are multiplied by a factor of x?

c) Predict the surface areas of these cubes:

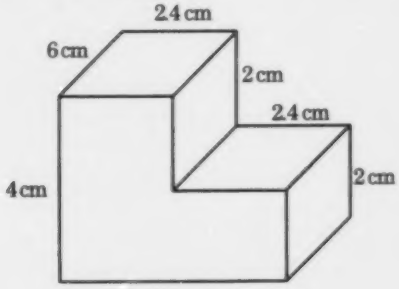




6. a) Find the surface area of this figure.

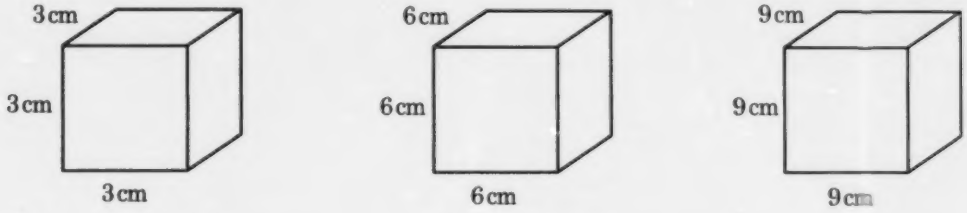


b) Predict the surface area of this figure.

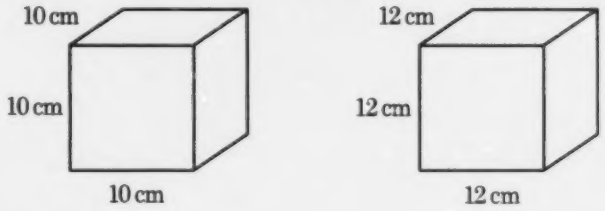


Exercise 3 (continued)

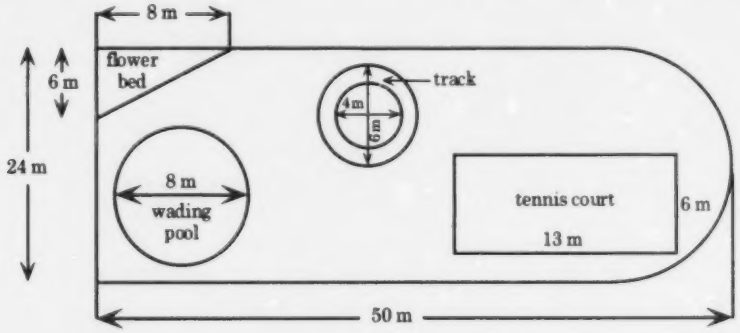
7. a) Find the volume of each cube.



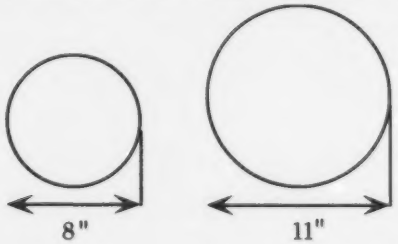
b) Predict the volume of each cube. Verify using the formula $V = lwh$



8. You are working for a contractor and need to determine the cost of placing sod on a playground shown below. Grass is to be placed everywhere except on the flower bed, the tennis court, the pool, and the track. There is to be grass inside the track. Calculate the total cost if the sod costs \$2.50/m^2. Show your work in an organized fashion. (The drawing is not to scale.)

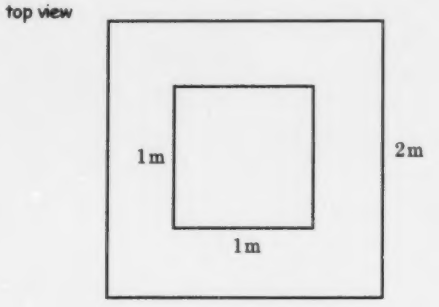


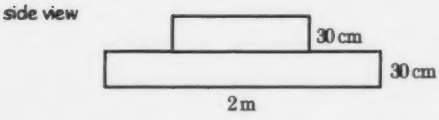
9. An 8-inch pizza is sold for \$6.50. Calculate the price that should be charged for an 11-inch pizza (of the same thickness) if the amount of pizza per dollar is to stay exactly the same.



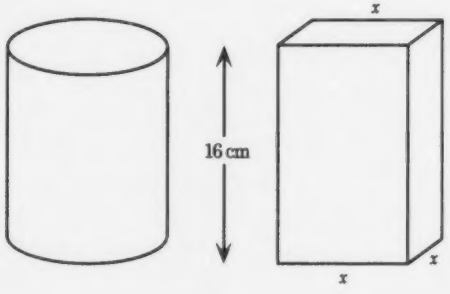
Exercise 3 (continued)

10. A conductor's platform is to be covered with a carpet. The diagrams below show the platform from the top and the side. The carpet has no pattern so it can be placed in any direction. Carpet is sold in rolls that are 3 m wide. How long of a piece of carpet do you need to buy to cover the entire platform if there are to be no seams on any flat surfaces? Draw a diagram of your cutting pattern. What percent of your purchase will be wasted?





11. You are on voluntary service in a developing country. You are a mathematician helping to develop a new company. The company will put oil in cans. The cans must be either cylindrical or square prisms. They must contain 1000 mL of liquid plus 100 mL of air space (that is the law). The height of the can must be 16 cm. Which can will require the least amount of material? How much less per can? If the company produces one million cans per year and tin costs $3.35 per square metre, how much would the company save in one year by using the cheaper design? If you get 5% of the savings as your commission, how much would you earn in commission for that year?



Volume of each
$= 1100 \text{ cm}^3$

Exercise 4

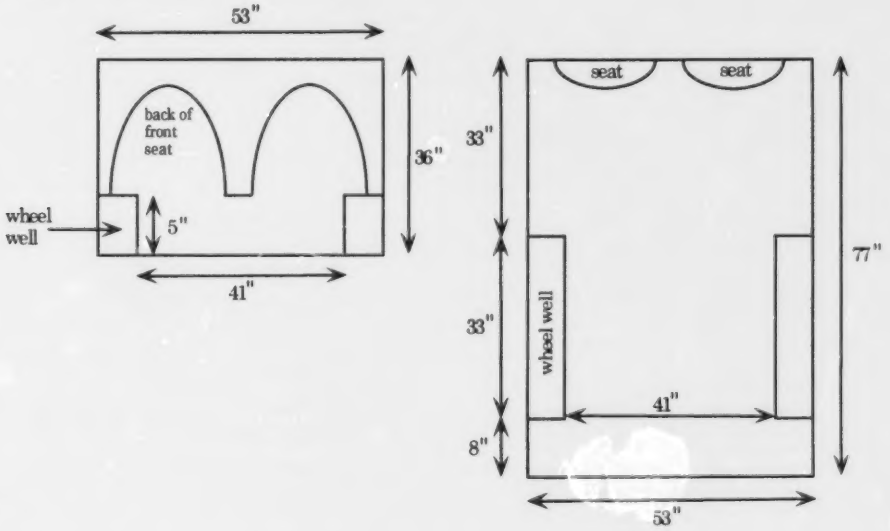
This section of questions tends to be of a longer, more difficult style. In some cases, students may need to make a scale model out of cardboard or another material. Solving the problem algebraically or mathematically may prove to be difficult.

1. The Robinsons ordered an entertainment unit from a store in a neighbouring city. They wanted to pick up the unit the next time they travelled to that city. The shipping department told them that the unit came in three boxes. The dimensions of the boxes were as follows:

box no.	width (inches)	length (inches)	height (inches)
1	24	64	24
2	20	56	18
3	18	79	3

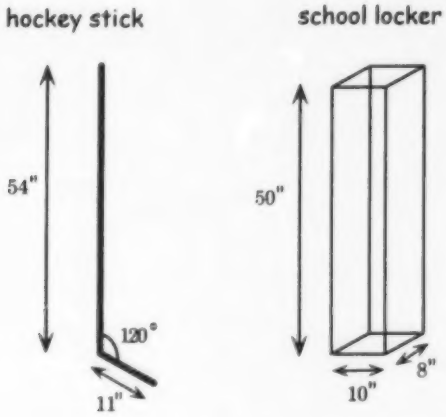
   a) Draw the three boxes. Indicate the scale used.

   b) The Robinsons own a sport utility vehicle, which when the back seats are down has a cargo area with the dimensions outlined below. Will the boxes fit in the space? You may have to construct a model out of cardboard.

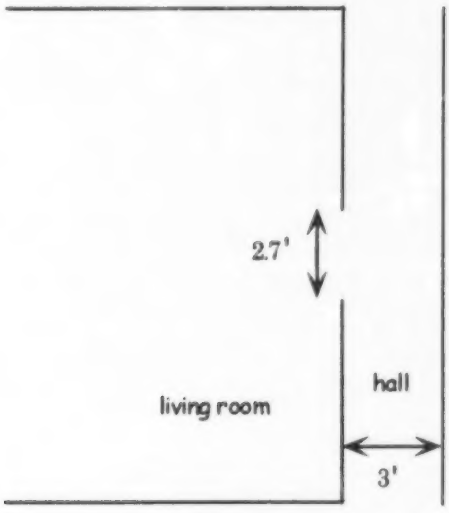


Exercise 4 (continued)

2. The dimensions of a hockey stick and a school locker are shown. The angle of the blade is 120°. Predict whether or not the stick will fit in the locker. Measure a stick and locker. Predict whether or not the stick will fit in the locker. Now test your prediction.



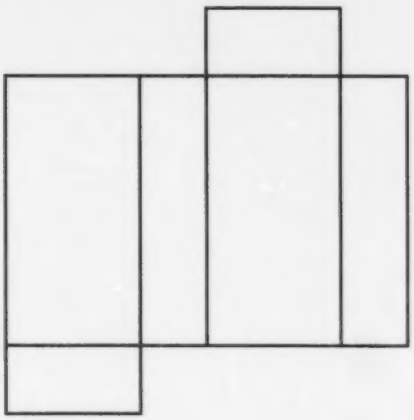
3. The McNeils are moving from one house to another. Their new house has a hallway as shown below. They have a couch that is 6'-2" long, 2'-10" wide, 1'-5" high at the front and 2'-9" high at the back. Draw a scale drawing of the hall and the couch. Will the couch go around the corner into the living room? You might have to make a scale model out of cardboard or do this question on a drawing program on a computer.



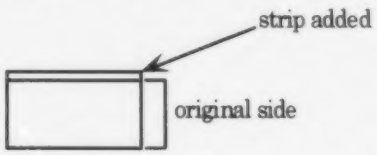
Exercise 4 (continued)

4. Shown below is a net of a cardboard box. Measure the net drawing. The scale is 1 cm = 10 cm.

   a) Assuming that no cardboard is needed to overlap, draw a 3-D scale drawing of the resulting box using the same scale. What is the surface area of the cardboard required? What is the volume of the real box?



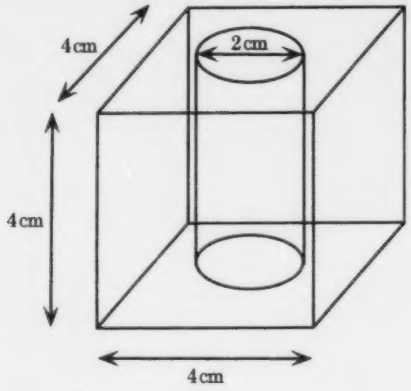
   b) Since the box has to be glued together, the net must have 2-cm strips added to the sides to allow for the overlapping portions where the glue is placed.



   c) Design a new net that would result in a box with the same volume but with 2-cm overlapping strips at each joint. What is this new surface area?

Exercise 4 (continued)

5. An ice-cube manufacturer makes ice cubes with cylindrical holes in them as shown.



a) What is the volume of ice in the cube?

b) The volume of ice decreases by 11% when it melts into water. What volume of water will be left when 10 cubes melt?

c) What is the total surface area including the inside of the hole of a single cube like this?

d) The manufacturer claims that these cubes cool a drink twice as fast as a regular cube with the same outside dimensions. Is this true if the claim is based on the assumption that a cube with a greater surface area cools faster?

e) What shape of cube might do a better job?

6. **Envelope Activity:**

Purpose: To communicate technical information orally, and to analyze geometric objects

Materials:
- 1 standard envelope
- scissors
- straightedge for folding

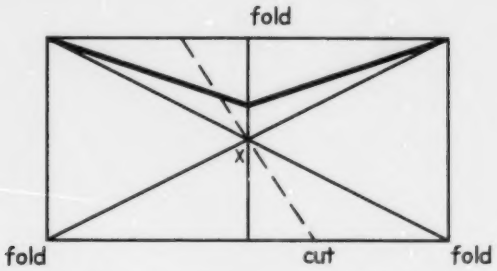
Organizaiton: Pairs — one person is the director, the other is the cutter

Directions

1. The director and cutter must communicate without seeing one another. Sitting back to back, the director describes to the cutter what to do. The cutter cannot see the diagram or the directions and the director cannot watch the cutter work. When the cutting is complete, they may both examine the product.

2. The director seals the envelope. The director then asks the cutter to fold along the diagonal lines and through the centre where the diagonals meet, as shown in the diagram. The director must describe these folds without showing the cutter the diagram and without looking at the envelope.

Exercise 4 (continued)



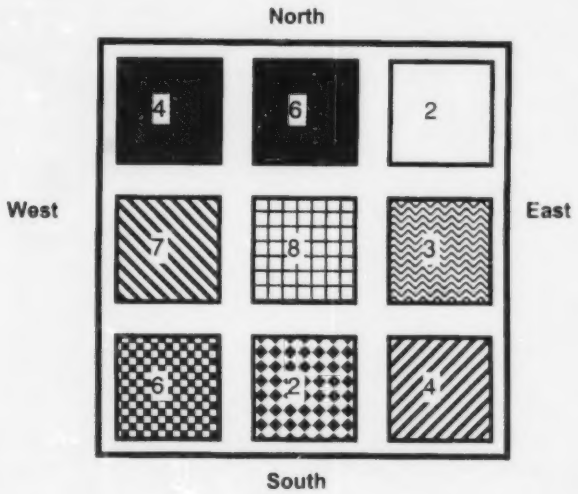
3. The director then asks the cutter to cut the envelope in half. The cut must be made from the top to the bottom through the point marked **X** (i.e., where the three folds meet), and may be on any angle. An example cut is shown as a dotted line. Both the cutter and director may now look at the two halves of the envelope.

4. Now that there are two halves, both the director and cutter try to decide what the object will look like if they popped it open into a 3-dimensional figure.

5. The final product is a tetrahedron (4-sided figure). Together the director and cutter determine what the ratio of width to length of an envelope must be to create a regular tetrahedron, i.e., all sides the same length and all faces equilateral triangles.

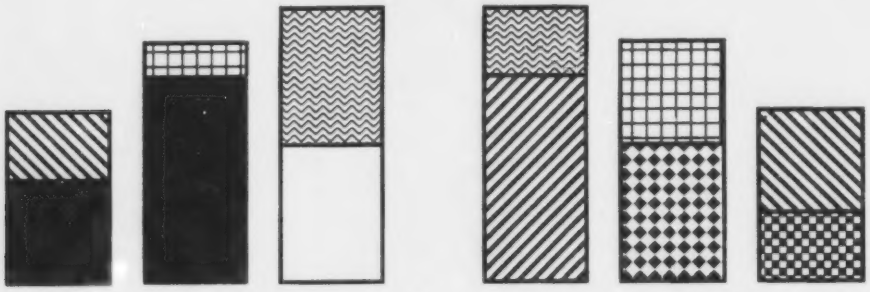
   Enrichment: Participants write a proof for this ratio.

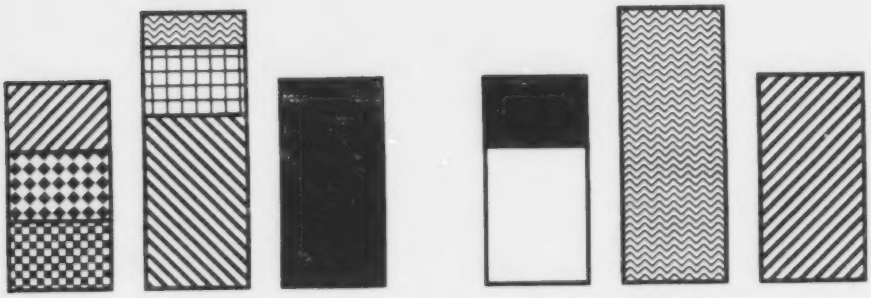
7. a) The diagram below shows the overhead view of a city block. Each building has a different pattern to distinguish it. The numbers on the top of the building give the number of stories in the building. You must draw the side view from each direction.



Exercise 4 (continued)

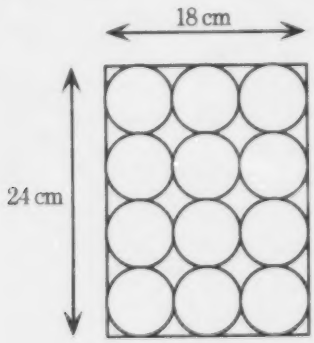
b) Given the following views, draw the overhead view and label the heights of the buildings. Each 1/4" is one story.





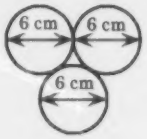
c) Make up one of each type of puzzle shown in parts (a) and (b). Ask your classmate to try to solve it.

8. a) Cans of soup are often packed in boxes as shown. Calculate the area of the box that is wasted in between all of the cans.



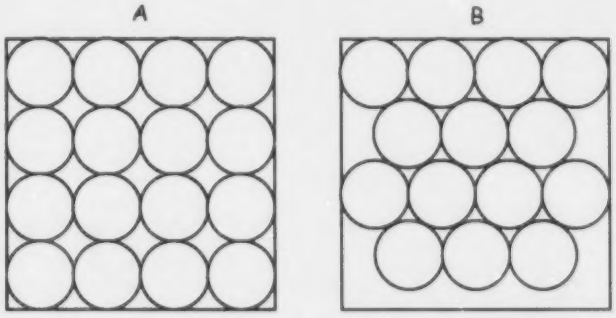
Exercise 4 (continued)

b) What is the area of the wasted portion in the centre of three cans packed tightly together?



You may need to know that the line connecting the centres of two touching circles passes through the point where they touch.

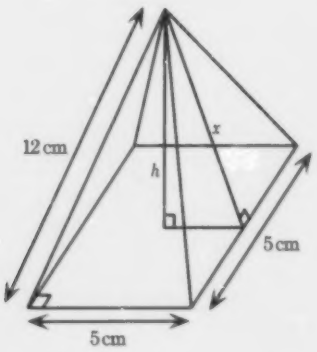
c) If the cans in part (b) were packed in square boxes, they could be packed in one of the two ways shown.



As you can see, type B leaves out two cans and does not fill the box so that the cans are tightly packed. These diagrams show a square packing box of 576 cm^2. Is there is a different-shaped carton whose base has an area of 576 cm^2 that actually increases the number of cans when packed as in type B? You may need to build or draw a scale model.

Exercise 4 (continued)

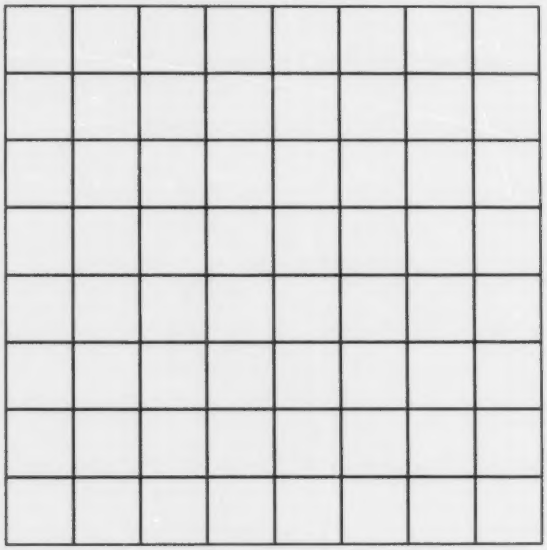
9. Calculate the volume of the pyramid shown. Use the formula $V = \frac{1}{3}Bh$ where B is the area of the base, and h is the altitude from the tip to the centre of the base. You will have to calculate h using the Pythagorean theorem.

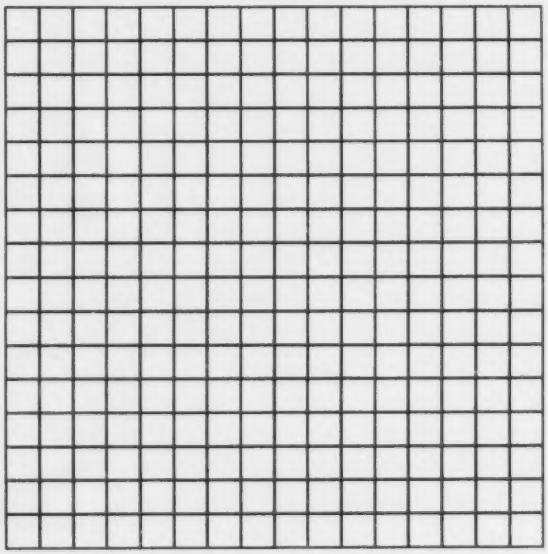


10. Buster the dog is tied to a 6-m chain that slides along a wire attached to two spiral spikes in the ground 14-m apart. What is the area of ground that Buster may cover? Draw the overhead view.

11. You are designing a can out of a sheet of metal that is 40 cm wide by 44 cm long. You are to make the largest can possible.
    a) Draw a diagram to show how this could be cut from the sheet.
    b) Explain why the diameter of the can must be 12.73 cm for it to be the biggest can possible.
    c) What height must the can be?
    d) What is the maximum volume of the can?

Attachment E-1





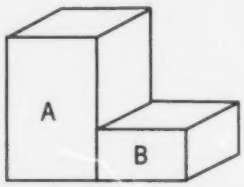
Unit E
2D/3D Projects
Solutions

Exercise 1 Solutions

1. a) $8 \times 12 \times 6 = 576\ cm^3$ b) $4 \times 4 \times 14\frac{3}{4} = 236\ in.^3$

2. Two possible ways are as follows:

   i) Splitting into two sections



   Volume A = $6 \times 3 \times 5 = 90\ cm^3$
   Volume B = $2 \times 3 \times 5 = 30\ cm^3$
   Total Volume = $120\ cm^3$

   ii) Volume of whole solid — volume of small solid cut-out
   $(6 \times 6 \times 5) - (3 \times 4 \times 5) = 120\ cm^3$

3. a) $155.5\ cm^3$ b) 577.3 cu. in. c) $100\ cm^3$ d) $58.54\ cm^3$ e) $90\ cm^3$

4. a) V = Volume solid – volume hole
   $= (8 \times 10 \times 22) - (\pi \times 1.5^2 \times 22)$
   $= 1604.5\ cm^3$

   b) V = Volume cylinder – volume solid
   $= \pi(1.75/2)^2(8.5) - (1.25 \times 0.75)(8.5)$
   = 12.48 cu. in.

5. Diameter of tube = 8 mm, volume of tube = 75 mL
   Volume of 1 cm length of toothpaste = $\pi(0.4)^2(1) = 0.5\ cm^2$
   Therefore, you can brush your teeth 75/0.5 = 150 times with the tube.

6. $\dfrac{10\ L}{0.500\ L/s} = 20$ seconds

   Note: The height of the pipe is not needed.

7. a) Rectangular:
   SA = $(2 \times 0.35 \times 5.5) + (2 \times 0.3 \times 5.5)$
   $= 7.15\ m^2$
   Capacity = $0.35 \times 0.3 \times 5.5$
   $= 0.5775\ m^3$

   Cylindrical:
   SA = $\pi(0.3)(5.5)$
   $= 5.18\ m^2$
   Capacity = $\pi(0.15)^2(5.5)$
   $= 0.39\ m^3$

Exercise 1 Solutions (continued)

b) Rectangular: $7.15 \times \$1.20/m^2 = \$8.58$
Cylindrical: $5.18 \times \$1.20/m^2 = \$6.22$

c) Rectangular: $\$8.58/0.5775 = \$14.86/m^3$ (better value)
Cylindrical: $\$6.22/0.39 = \$15.94/m^3$

8. The basic assumption is that the capacity of the pipe must be emptied of its cold water before the hot water arrives.

Capacity of the pipe is 55 seconds x 20 mL/sec = 1100 mL = 1100 cm^3

Vol. pipe $= \pi(1.75/2)^2 h$ cm^3 h cm = length (height) of pipe
1100 $= \pi(1.75/2)^2 h$

$$h = \frac{1100}{\pi(1.75/2)^2} = 457.3$$

Length of pipe = 457.3 cm

9. Volume of a cord $= 4 \times 4 \times 8 = 128$ cu. ft.

Volume of truck box $= l \times w \times h$
$128 = 4.5 \times 7.25 \times h$ Height of wood = h ft.

$$h = \frac{128}{4.5 \times 7.25} = 3.92$$

Height of wood is at least 4 ft.

10. $4 \times (4 \times 4 \times 42) = 2688$
$24 \times (1 \times 4 \times 36) = 3456$
$6 \times (2 \times 4 \times 48) = 2304$
Total volume = 8448 cu. in.

Total board feet = 8448 / 144 = 58.67 board feet
Cost = 58.67 x \$2.25 = \$132.01
PST and GST = \$18.48
Total cost = \$150.49

Exercise 2 Solutions

1. a) 523.6 cm^3 b) 1767.1 cubic inches or 1.02 cubic feet c) 745.5 cm^3

2. a) 140.2 cm^2 b) 2642.1 square inches or 18.3 square feet c) 620.2 cm^2

3. a) V = 452.4 cm^3, SA = 339.3 cm^2 (incl bottom) b) V = 8182.77 cubic inches or 4.7 cubic feet, SA = 2337.9 sq. inches or 16.24 sq. feet

4. a) Assume 1.6 km = 1 mile
   6.37×10^6 m =6370 km = 3981.25 mi.
   i) SA = 1.99×10^8 sq. mi.
   ii) SA = 5.1×10^8 km^2

   b) Volume of Earth = 2.64×10^{11} cu. mi. Volume of Earth = 1.08×10^{12} km^3

5. Radius = 2: V =33.5 cm^3, SA = 50.3 cm^2
   Radius = 4: V = 268.1 cm^3, SA = 201.1 cm^2
   These calculations show that doubling a sphere's radius multiplies its volume by 8, and its surface area by 4.

6. Diameter = 4 m, V = 33.5 m^3
   New volume = 33.51 + 30 = 63.5 m^3
   New radius :

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r = \sqrt[3]{\frac{3(63.5)}{4\pi}} = 2.48 \text{ m}$$

   New diameter = 4.96 m
   New surface area = 77.3 m^3

7. The rate of expansion is $\frac{\text{volume at 5"}}{\text{5 seconds}}$

$$= \frac{\frac{4}{3}\pi 5^3}{5}$$

$$= 104.7 \text{ cubic inches / second}$$

The volume at $r = 10"$ is $\frac{4}{3}\pi r^3$, $V = \frac{4}{3}\pi 10^3$

$= 4188.8$ cubic inches

Time taken $= \frac{4188.8 \text{ cubic inches}}{104.7 \text{ cubic inches / s}}$ = 40 seconds from start, or 35 more seconds

Exercise 2 Solutions (continued)

8. $SA_{top} = \frac{4\pi 3^2}{2} = 56.5 \text{ m}^2$

SA_{middle} = circumference x height
= 6π x 8
= 150.8 m^2

SA_{bottom} = πrs $\qquad s = \sqrt{5^2 + 3^2}$
= $\pi(3)(5.83)$
= 54.955 m^2

SA_{total} = 262.255 m^2

Total litres needed = 262.255 ÷ 2.5 = 104.902 (need 105 cans minimum)

Cost = 105 x \$9.50 x 1.14 = \$1137.15

Exercise 3 Solutions

1. a) i) 28.27 cm^2 ii) 113.10 cm^2 iii) 254.47 cm^2

   b) When the radius doubles, the area increases four times.
   (113.10 ÷ 28.27 = 4)
   When the radius triples, the area increases nine times

   c) If the area of the original is A and the factor is x, the new area $A' = Ax^2$. You multiply the area by x^2.

   d) Students may have trouble identifying the multiplication factor x.
   i) $x = 12 \div 3 = 4$, $A' = 452.39\ cm^2$
   ii) $x = 21 \div 3 = 7$, $A' = 1385.94\ cm^2$
   iii) $x = 4 \div 3 = 4/3$, $A' = 50.27\ cm^2$
   iv) $x = 10 \div 3 = 10/3$, $A' = 314.16\ cm^2$

2. a) i) 9 cm^2 ii) 36 cm^2 iii) 81 cm^2

   b) Area is multiplied by 4. Area is multiplied by 9.

   c) Area is multiplied by x^2.

   d) i) Factor is $\frac{12}{3} = 4$. New area $= 9 \times 16 = 144\ cm^2$.

   ii) Factor is $\frac{21}{3} = 7$. New area $= 9 \times 49 = 441\ cm^2$.

   iii) Factor is $\frac{4}{3}$. New area $= 9 \times \left(\frac{16}{9}\right) = 16\ cm^2$.

   iv) Factor is $\frac{10}{3}$. New area $= 9 \times \left(\frac{100}{9}\right) = 100\ cm^2$.

3. a) In this question students will have to follow the same pattern they saw in questions 1 and 2 to create the steps to solve this problem.

   A suggested pattern is as follows:

base = 3 cm	height = 3 cm	area = 4.5 cm^2
base = 6 cm	height = 6 cm	area = 18 cm^2
base = 9 cm	height = 9 cm	area = 40.5 cm^2

   b) Area of each triangle is 1 sq. in. Students should realize that the shape of the triangle does not alter its area.

   When base and height are both doubled, the new area is the original area multiplied by 4. When base and height are both tripled, the new area is the original area multiplied by 9. In general, if the base and height are both multiplied by x, then the original area is multiplied by x^2.

Exercise 3 Solutions (continued)

c) A suggested pattern is as follows:

base = 3 cm	height = 3 cm	area = 4.5 cm^2	
base = 6 cm	height = 3 cm	area = 9 cm^2	(area doubles)
base = 9 cm	height = 3 cm	area = 13.5 cm^2	(area triples)
base = 3 cm	height = 3 cm	area = 4.5 cm^2	
base = 3 cm	height = 6 cm	area = 9 cm^2	(area doubles)
base = 3 cm	height = 9 cm	area = 13.5 cm^2	(area triples)

When the base is multiplied by x and the height by y, the original area is multiplied by xy.

4. a) Students should suggest that the smaller grid be used to make a better estimation. (The larger grid has squares that are 1 cm^2, whereas the smaller grid has squares that are 0.25 cm^2.)

Areas of blobs are approximate:

i) 19 large squares or 75 small squares (equals 75 ÷ 4 = 18.75 large squares)

ii) 21 large squares or 78 small squares (equals 78 ÷ 4 = 19.5 large squares)

b) i) 6 large squares (6 cm^2), 31 small squares (7.75cm^2)

ii) 8 large (8 cm^2), 32 small (8 cm^2)

iii) 5 large (5 cm^2), 19 small (4.75 cm^2)

c) i) Sides are 3.7 cm, 4.1 cm, and 5.6 cm

$$S = \frac{3.7 + 4.1 + 5.4}{2} = 6.6$$

$$A = \sqrt{6.6(6.6 - 3.7)(6.6 - 4.1)(6.6 - 5.6)}$$

$$A = 7.58 \text{ cm}^2$$

ii) Sides are 4.0 cm, 4.5 cm, and 4.5 cm

$$A = \sqrt{6.5(6.5 - 4.0)(6.5 - 4.5)(6.5 - 4.5)}$$

$$A = 8.06 \text{ cm}^2$$

iii) Sides are 2.0 cm, 4.5 cm, and 5.7 cm

$$A = \sqrt{6.1(6.1 - 2.0)(6.1 - 4.5)(6.1 - 5.7)}$$

$$A = 4.00 \text{ cm}^2$$

d) If sides are doubled, Heron's formula yields the following:

i) $A = \sqrt{13.2(13.2 - 7.4)(13.2 - 8.2)(13.2 - 10.8)}$

$A = 30.31 \text{ cm}^2$

New area is four times the original area.

Exercise 3 Solutions (continued)

ii) $A = \sqrt{13.0(13.0-8.0)(13.0-9.0)(13.0-9.0)}$

$A = 32.25 \text{ cm}^2$

iii) $A = \sqrt{12.2(12.2-4.0)(12.2-9.0)(12.2-11.4)}$

$A = 16.00 \text{ cm}^2$

5. a) i) $SA = 6(3 \times 3) = 54 \text{ cm}^2$

ii) $SA = 216 \text{ cm}^2$

iii) $SA = 486 \text{ cm}^2$

b) When sides are doubled, SA is multiplied by 4.

When sides are tripled, SA is multiplied by 9.

When sides are multiplied by x, SA is multiplied by x^2.

c) New area = $54(10/3)^2 = 600 \text{ cm}^2$ New area = $54(12/3)^2 = 864 \text{ cm}^2$

6. a) $SA = 33.6 \text{ cm}^2$

b) Since all sides are doubled, new surface area = $33.6(2)^2 = 134.4 \text{ cm}^2$.

7. a) $V = 3 \times 3 \times 3 = 27 \text{ cm}^3$

$V = 6 \times 6 \times 6 = 216 \text{ cm}^3$ (when sides are doubled, volume is multiplied by 8)

$V = 9 \times 9 \times 9 = 729 \text{ cm}^3$ (when sides are tripled, volume is multiplied by 27)

Rule: New volume = original volume multiplied by x^2.

b) 10 cm cube: Volume = $27(10/3)^3 = 1000 \text{ cm}^3$

12 cm cube: Volume = $27(12/3)^3 = 1728 \text{ cm}^3$

8. Total area = area rectangle + area semicircle (radius semicircle = 12 m)

= ((50– 12) x 24) + $\pi(12)^2/2$

= 912 + 226.2 = 1138.2 m^2

Subtract area not to be covered with sod:

flower bed: $6 \times 8 \div 2 = 24 \text{ m}^2$

pool: $\pi 4^2 = 50.3 \text{ m}^2$

court: $13 \times 6 = 78 \text{ m}^2$

track: $\pi 3^2 - \pi 2^2 = 15.7 \text{ m}^2$

Total area to be subtracted = 24.0 + 50.3 + 78.0 + 15.7 = 168.0 m^2

Total area to be covered = 1138.2 m^2 – 168.0 m^2 = 970.2 m^2

Total cost = 970.2 x \$2.50 = \$2425.50

Cost includes GST and PST = \$2765.07

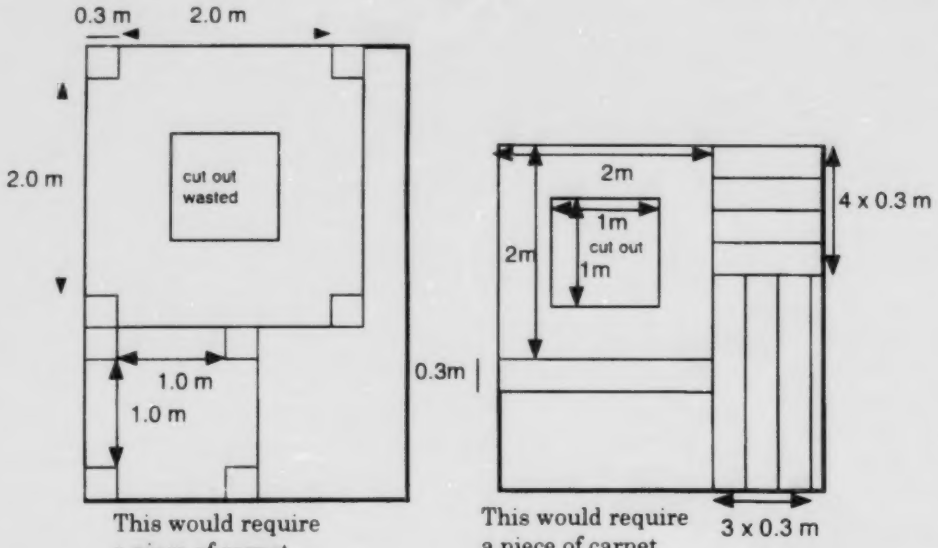
Exercise 3 Solutions (continued)

9. Area of small pizza = $\pi 4^2 = 50.27$ sq. in.
Cost per sq. in. = \$6.50 ÷ 50.27 = \$0.1293/sq. in.

Area large pizza = $\pi(5.5)^2 = 95.03$ sq. in.
Price charged = 95.03 x \$0.1293 = \$12.29

Discuss with students why volume does not have to be considered.

10. Students may think of these two solutions. Others may be possible.



This would require a piece of carpet that is 3 m wide and 4.2 m long

This would require a piece of carpet that is 3 m wide and 3.2 m long

The total area to be covered = (length x width) + 4 small edges + 4 large edges
= (2 x 2) + 4(1 x 0.3) + 4(2 x 0.3)
= 4 + 1.2 + 1.2
= 6.4 m^2

Solution 1 uses 3 x 4.2 m = 12.6 m^2

Percent used = $\frac{6.4}{12.6} \times 100 = 51\%$ used or 49% wasted

Solution 2 uses 3 x 3.2m = 9.6 m^2

Percent used = $\frac{6.4}{9.6} \times 100 = 66\%$ used or 33% wasted

There may be a more efficient way to cut the carpet.

Exercise 3 Solutions (continued)

11. Volume = 1100 mL = 1100 cm^3

Square prism solution:

V = lwh

(Let the lengths of sides of square be x cm.)

$1100 = (x)(x)(16)$

$$x^2 = \frac{1100}{16}$$

$x = 8.29$

Length of side = 8.29 cm

Surface area:

$= 4(16 \times 8.29) + 2(8.29)^2$ cm^2

$= 668$ cm^2

Cylindrical solution:

V = $\pi r^2 h$

(Let the radius of the circle be r cm.)

$1100 = \pi r^2(16)$

$$r^2 = \frac{1100}{16\pi}$$

$r = 4.68$

Radius = 4.68 cm

Surface area:

$= 2\pi(4.68)^2 + 2\pi(4.68)(16)$ cm^2

$= 608.1$ cm^2

The cylindrical can requires 59.9 cm^2 less material. (= 0.005 99 m^2).

If the company used the cylindrical can they would save

1 000 000 cans x 0.005 99 m^2 / can = 5990 m^2

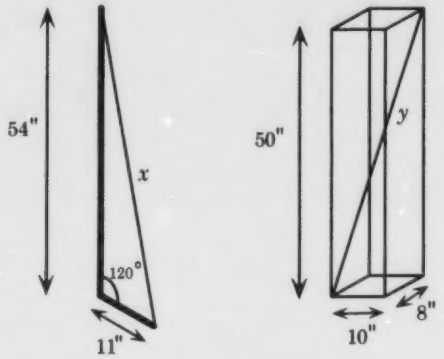
5990 m^2 x \$3.35/m^2 = \$20 066.50 in that year

Your commission on this would be \$20 066.50 x 5% = \$1003.33.

Exercise 4 Solutions

1. The total volume of the truck's rear area is enough to hold the boxes. However, it is noted that the 79" long box will have to sit in between the front seats. This is an actual problem from someone's experience. The boxes do fit. A scale model is likely necessary.

2. Mathematics will be able to solve this problem, however, the width of the stick handle and blade may cause problems.



In this problem, the distance x must be less than the distance y.

Use the cosine law:

$a^2 = b^2 + c^2 - 2bc \cos A$

$x^2 = 54^2 + 11^2 - 2(54)(11)\cos 120°$

$x^2 = 3631$

$x = 60.26"$

First find the distance across the diagonal of the locker:

$d^2 = 10^2 + 8^2$ or $y^2 = 10^2 + 8^2 + 50^2$

$d = 12.81"$ $y^2 = 2664$

Now use this to find y:

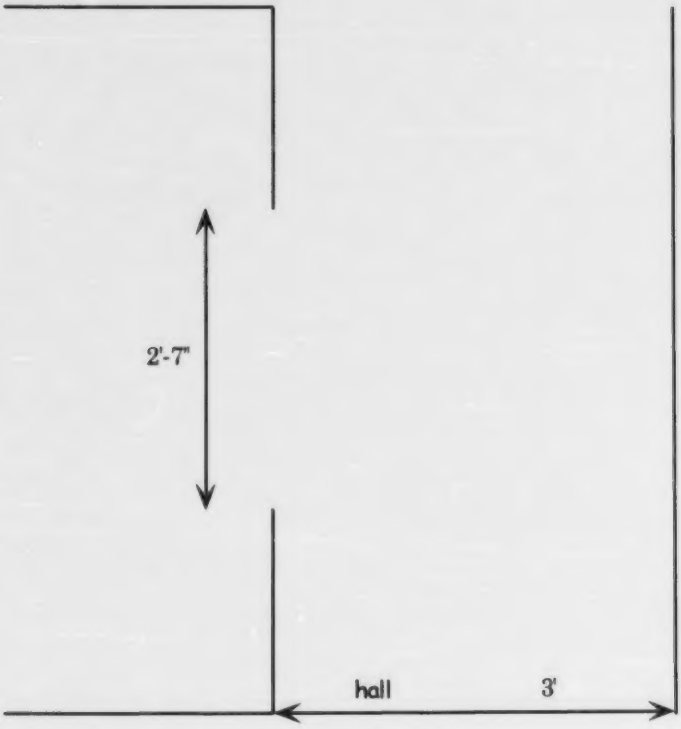
$y^2 = 50^2 + 12.81^2$

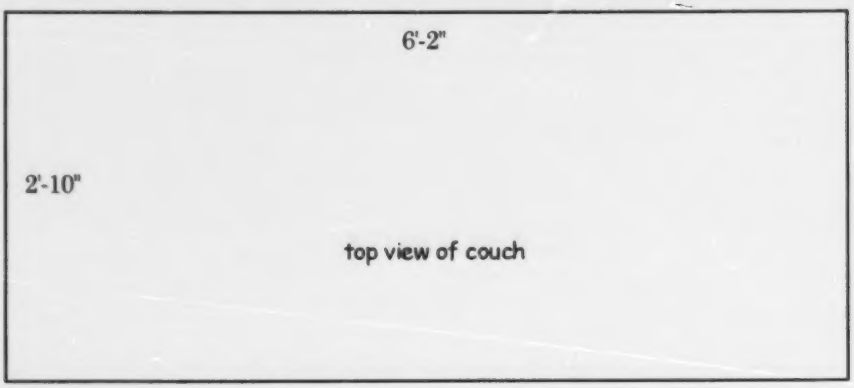
$y = 51.61"$

No, this stick will not fit into this locker. Have students try different-sized lockers or try one of their own in the school. Use a real hockey stick too. How can you account for the width of the handle and the blade?

3. On the next page is a scale drawing of both the hallway and the couch. The scale is 1' = 1". Students may cut out the couch and try to get it through the opening. Without a scale model of the couch, it is not obvious whether or not the couch will pass through the opening. Make three-dimensional models of the hallway and couch, and investigate! The height and width of the couch are also restrictions.

Exercise 4 Solutions (continued)

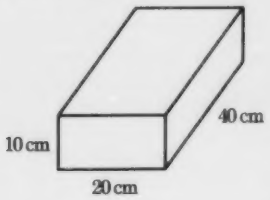




Exercise 4 Solutions (continued)

4. a) The real box would have the following dimensions:

20 cm x 10 cm x 40 cm



Surface area

$2 \times 10 \times 20 = 400$

$2 \times 40 \times 10 = 800$

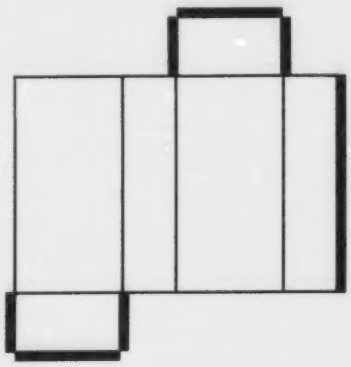
$2 \times 20 \times 40 = 1600$

Total = 2800 cm^2 or 0.28 m^2

Volume = $10 \times 20 \times 40$

= 8000 cm^3 or 0.08 m^3

b) A new possible net is



New surface area is: old SA + SA of strips added

$= 2800 + 4(2 \times 10) + 2(2 \times 20) + (2 \times 40)$

$= 2800 + 80 + 80 + 80$

= 3040 cm^2 or 0.304 m^2

5. a) Volume of ice = volume of cube – volume of cylinder

$= (4 \times 4 \times 4) - (\pi(1)^2 \times 4)$

$= 64 - 12.57$

= 51.43 cm^3

b) Each cube loses 11% of 51.43 cm^3 = 5.66 cm^3

Water produced by 1 cube = 51.43 – 5.66 = 45.77 cm^3 of water

10 cubes produce 10×45.77 = 457.7 cm^3 of water

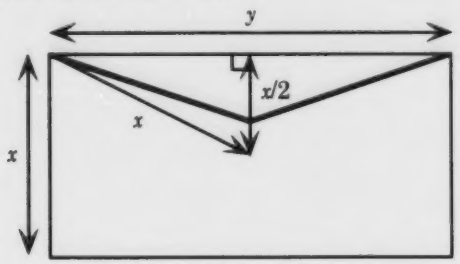
Exercise 4 Solutions (continued)

c) The total surface area can be found by
$SA_{cube} - SA_{hole\ on\ both\ ends} + SA_{cylinder\ inside}$
$= 6(4 \times 4) - 2(\pi(1)^2) + (2\pi 4)$
$= 96 - 6.28 + 25.13$
$= 114.85\ cm^2$

d) The claim would be true if the surface area of this cube is twice that of a 4 x 4 x 4 cube. The SA of a 4 x 4 x 4 cube is 96 cm^2.
114.85 ÷ 96 = 1.2. This cube will cool a drink only 1.2 times as fast. Therefore, the manufacturer's claim is not valid.

e) One possibility includes creating a similar cube, except inside the cube instead of having a cylinder, there is a square prism of sides 2 cm long.

6. The final product is a tetrahedron. To get a regular tetrahedron (all sides the same length) the ratio of width to length must be as shown below.



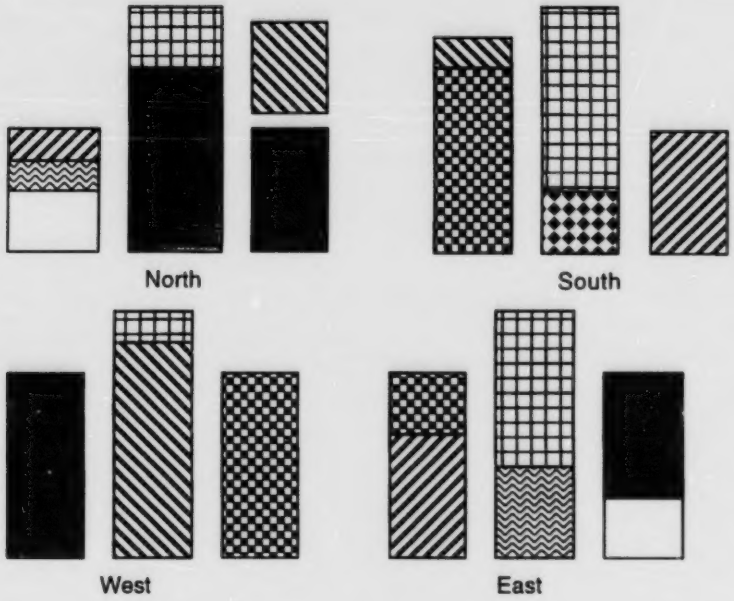
Let y be the length of the envelope. For a regular tetrahedron to appear, the distances marked x must have the same length. Now use the Pythagorean theorem.

$$x^2 = (\frac{x}{2})^2 + (\frac{y}{2})^2$$
$$x^2 = \frac{x^2}{4} + \frac{y^2}{4}$$
$$\frac{3x^2}{4} = \frac{y^2}{4}$$
$$3x^2 = y^2$$
$$3 = \frac{y^2}{x^2}$$
$$\frac{y}{x} = \sqrt{3}$$

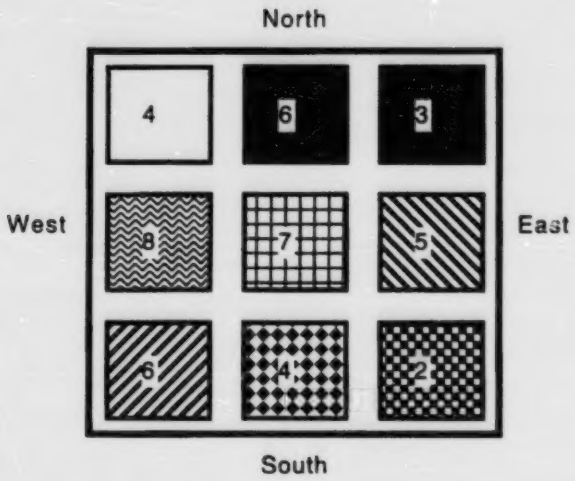
For a regular tetrahedron to appear, the length must be $\sqrt{3}$ times as long as the width.

Exercise 4 Solutions (continued)

7. a)



b)



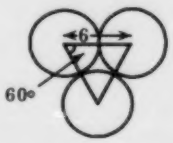
Exercise 4 Solutions (continued)

8. a) Area of box is 24 cm x 18 cm = 432 cm^2
The diameter of each can is 18 ÷ 3 = 6 cm
Thus the radius of each can is 3 cm.

The area of one can is πr^2 or $\pi(3)^2 = 28.27$ cm^2
The total area of the cans is 12 x 28.27 = 339.29 cm^2

The total area wasted is 432 – 339.29 = 92.71 cm^2

b)



Begin by adding to the diagram lines connecting the centres of the circles. These are also 6 cm since they are 2 x radius. This forms an equilateral triangle, which has angles of 60°.

The area wasted is the area of the equilateral triangle — 3 x area of the pie shaped piece
Area triangle: *Use Heron's formula or Pythagoras*

$= 15.59$ cm^2

Area pie shaped pieces $= \frac{60}{360}$ or $\frac{1}{6}$ area circle × 3 pieces

$= \frac{\pi(3)^2}{6} \times 3$

$= 14.14$ cm^2

Area wasted is 15.59 – 14.14 = 1.45 cm^2

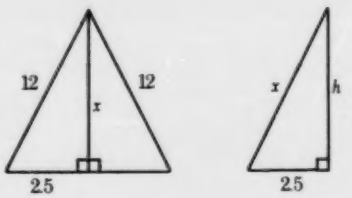
c) Students may wish to experiment with pennies and using cartons of various shapes and sizes.

9. To solve for h, you first need to know the "height," x, of the face of one of the sides.

$12^2 = (2.5)^2 + x^2$
$x = 11.74$ cm

Now use this to find the height, h, of the pyramid

$11.74^2 = (2.5)^2 + h^2$
$h = 11.47$ cm



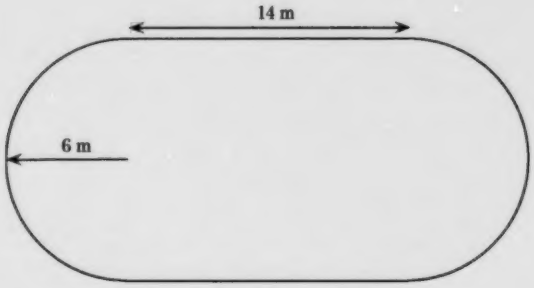
Volume of pyramid $= \frac{1}{3}$ base area × height

$= \frac{1}{3}(5 \times 5) \times 11.47$

$= 95.58$ cm^3

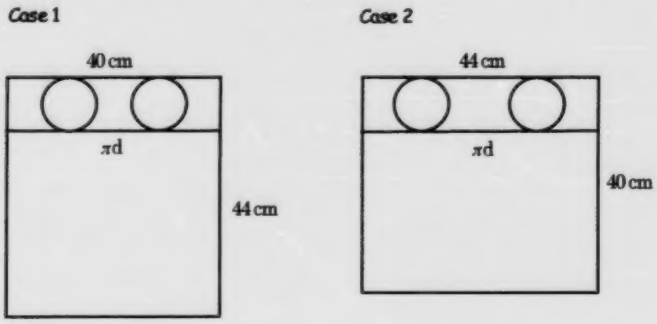
Exercise 4 Solutions (continued)

10. The area that the dog can cover is a rectangle with semi circles on the ends as shown.



Area $= (14 \times 12) + (\pi 6^2)$
$= 281.1\ m^2$

11. a) Here are two possible nets:



Case 1

b) Assuming that the width of the sheet (40 cm) is the circumference of the can (πd) then the diameter (d) is given by:

$\pi d = 40$
$d = 40 \div \pi = 12.73$ cm

c) Since the diameter of the can must be 12.73 cm, the height of the can must be $44 - (12.73) = 31.27$ cm.

d) This gives a volume of

$V = \pi r^2 h$
$= \pi(12.73/2)^2\ (31.27)$
$= 2354.6\ cm^3$ or 2.354 L
$= 3979.92\ cm^3$ or 3.979.92 L

Exercise 4 Solutions (continued)

Case 2

b) Assuming that the length of the sheet (44 cm) is the circumference of the can (πd), then the diameter d is given by $\pi d = 44$.

Therefore, $d\upsilon = 44 \div \pi = 14.01$ cm.

c) Since the diameter of the can must be 14.01 cm, the height is $40 - 14.01 = 25.99$ cm.

d) This gives a volume of $V = \pi r^2 h$.

$V = \pi(14.01 \div 2)^2 (25.99) = 4006.57$ cm $= 4.007$ L

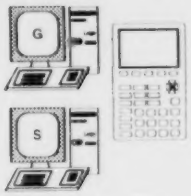
Conclusion: Case 2 yields the greater volume.

Unit F
Relations and Functions

Exercise 1

1. A dog suddenly runs out into the street in front of a car. The table below shows stopping distances for a car at different speeds, assuming the road surface is good and the driver is alert.

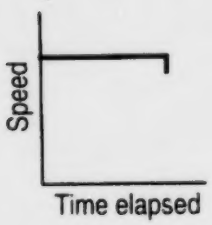
Speed (km/h)	0	20	40	60	80	100	120
Stopping distance (metres)	0	7.5	20	37	60	88	120

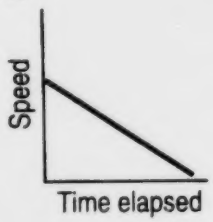


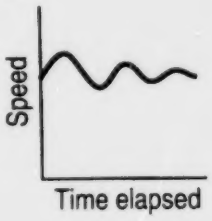
a) Which of the two variables should be the independent variable and which the dependent variable?
b) Draw a graph to represent this function.
c) Does it make sense to draw a smooth curve through the points? Why?
d) Use the graph to estimate the stopping distance for a car travelling at
   i) 50 km/hr
   ii) 90 km/hr
   iii) 150 km/hr
   How accurate is the estimate in each case?

2. Indicate which graph matches the statement.

a) A train pulls into a station and lets off its passengers.

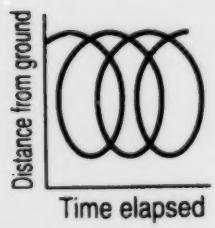
i)


ii)


iii)


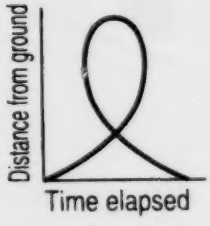
iv)

b) A man takes a ride on a ferris wheel.

i)


ii)

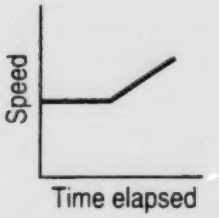
iii)

iv)


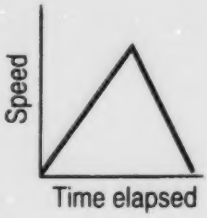
Exercise 1 (continued)

c) A woman climbs a hill at a steady pace and then starts to run down one side.

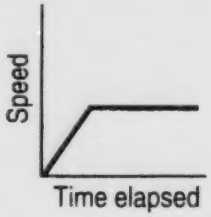
i)



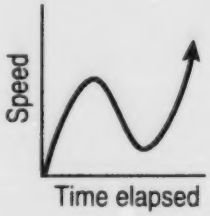
ii)



iii)



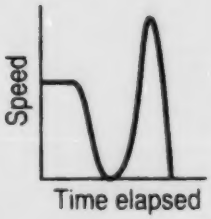
iv)



3. Describe briefly what each graph could represent.

a)

b)



4. Sketch a reasonable graph for each of the following statements.
   a) The altitude of a football depends on the number of seconds since it was kicked.
   b) The temperature of your coffee is related to how long it has been cooling.
   c) You turn on the hot water faucet. As the water runs, its temperature depends on the number of seconds it has been since you turned on the faucet.
   d) As you play with a yo-yo, the number of seconds that have passed and the yo-yo's distance from the floor are related.
   e) You pour some popcorn into a popper and turn it on. The number of pops per second depends on how long the popper has been turned on until all the popcorn is popped.

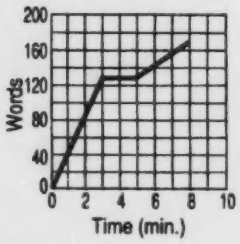
Exercise 1 (continued)

5. Richard types for three minutes at the rate of thirty-five words per minute. He takes a two-minute break and then types for three more minutes at the rate of twenty words per minute.

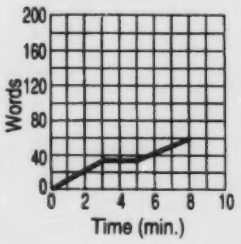
   a) The graphs below show time on the horizontal axis and the number of words typed on the vertical axis. Which graph represents Richard's time at the keyboard?

Richard's Typing

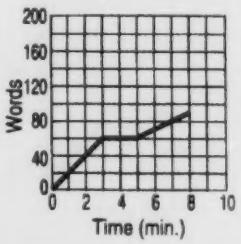
i)



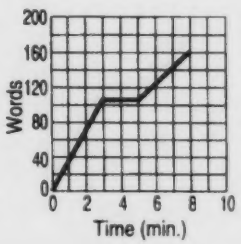
ii)



iii)



iv)



   b) Choose one of the answers in (a) that was not correct and compose a problem having that answer as its corresponding graph.

6. Draw graphs to represent the following statements.

   a) Sara walks from her home to the store. Halfway to the store, she realizes that she forgot to bring money, so she turns around, returns home, gets her money, and then walks all the way to the store. Graph time on the horizontal axis and distance from home on the vertical axis.

   b) Rashid is jumping on the trampoline. Graph time on the horizontal axis and his distance off the ground on the vertical axis.

   c) Kendra is speeding along the highway and is stopped by a police officer. The officer gives her a ticket and then she continues on her way. Graph time on the horizontal axis and her speed on the vertical axis.

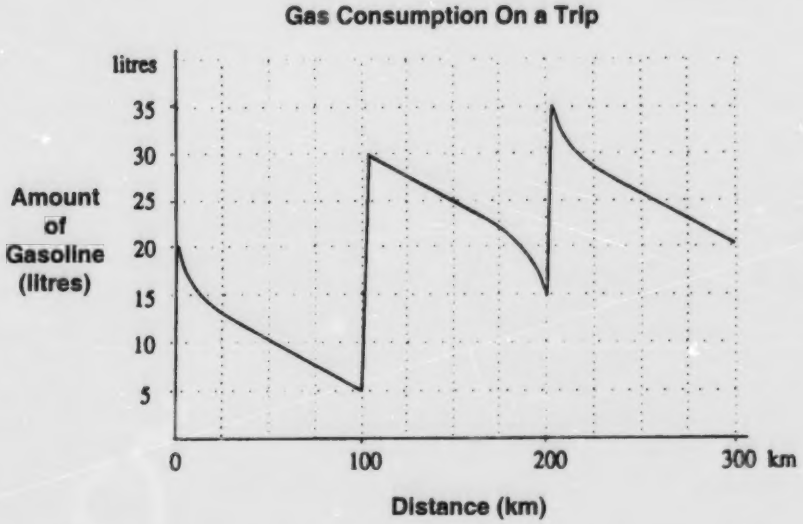
   d) Carlos lives in a large city and travels to school on a local bus that stops at every block to let passengers on and off.

      i) Graph time on the horizontal axis and the speed of the bus on the vertical axis.

      ii) Graph time on the horizontal axis and the distance Carlos has travelled on the vertical axis.

Exercise 1 (continued)

7.



The above graph shows the amount of gasoline in the gas tank during a recent trip.

a) How much gas does the gas tank hold?

b) How many times did the driver "fill up"? Where were those locations?

c) Where was the most gas bought?

d) How much gas was used for the entire trip?

e) If no stops had been made for gas fill ups, where would the car have run out of gas?

f) Town driving uses more gasoline than country driving because you have to keep stopping and starting. Was this driving in town or in the country?

g) Where might the town(s) be located on this journey?

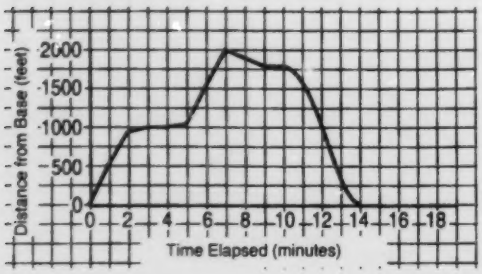
h) Does the graph suggest that the gas stations were in the country, in the centre of a town, or on the edge of a town?

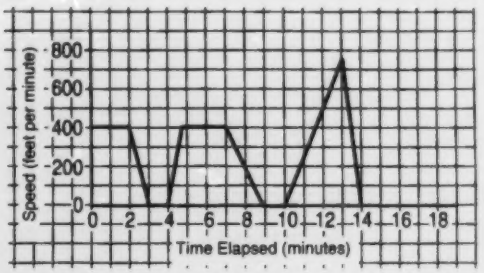
i) Sketch a graph to show what would have happened if the car had only stopped at the first gas station.

Exercise 1 (continued)

8. The Mount Southington Ski Resort claims its lift travels at 400 feet per minute and is 2000 feet long. The graphs that follow record in two ways the trip of one skier riding up the mountain on the lift and then skiing down the mountain. One graph represents the distance from the bottom of the mountain as a function of time, the other represents the speed of the skier as a function of time.

Skiing





a) Did the chairlift stop before the skier reached the top of the mountain? Explain your answer.

b) Is it possible to tell from the distance graph that the speed has dropped to zero? Explain your answer.

c) What is the top speed of the chairlift in miles per hour? (1 mile = 5280 feet)

d) What is the top speed of the skier in miles per hour?

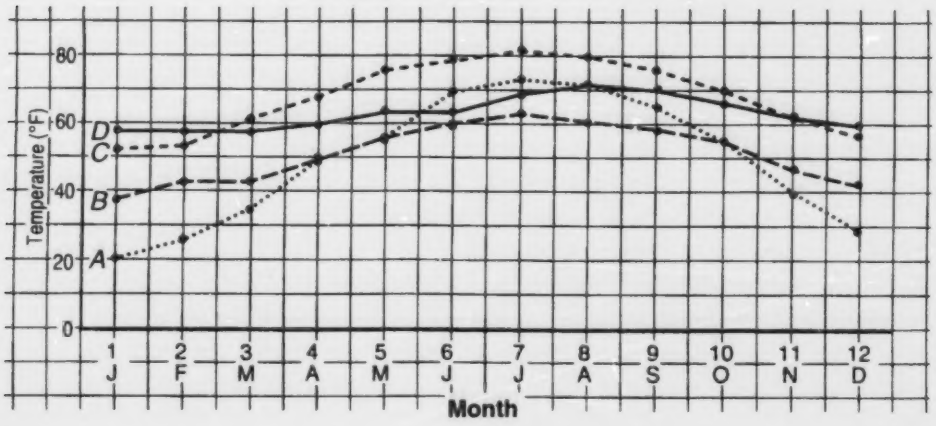
e) Notice that between the ninth and tenth minute the speed is 0. What do you think happened?

f) Label five points ***A***, ***B***, ***C***, ***D***, and ***E*** on each graph, correlating distance and speed based on time elapsed. Explain why and how these points are related and what is happening at each time.

Exercise 1 (continued)

9. In the graph below you will find the average monthly temperatures for Jacksonville, Florida; Seattle, Washington; San Diego, California; and Chicago, Illinois. These temperatures are based on records for the thirty-year period from 1951 to 1980, inclusive. On the basis of what you know about the climate of each state, guess which state is associated with each graph.

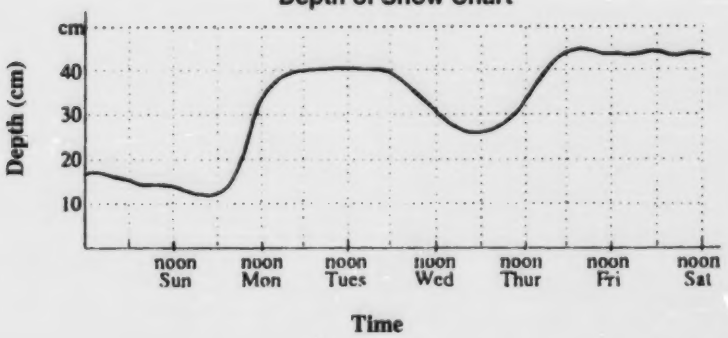
Average Temperatures in Four Cities



a) State A is _____. b) State B is _____.
c) State C is _____. d) State D is _____.

10. The manager of a ski lodge keeps a record of the depth of snow outside the lodge throughout the ski season. The graph below shows data collected during one week last winter.
    a) On which days during this week did it snow?
    b) On which day did the greatest amount of snow fall?
    c) Can you estimate from the graph the total depth of new snow that fell during the week?
    d) One day that week, it rained all day. Which day was this?
    e) Estimate the amount of snow that melted on the rainy day.
    f) If the snow had continued to melt at the same rate for one more day, what depth would have been left at the end of the day?

Depth of Snow Chart



Exercise 2

1. How would you use functional notation to express the following statements? (Use your choice of notation in each case, but explain clearly what the symbols mean.)
   a) Your hunger depends on the length of time since your last meal.
   b) The number of people suffering from asthma on any day in a city is related to the level of pollen in the atmosphere.

2. It is claimed that the formula $f(T) = 7.2T + 88$ is a model for the relationship between the temperature in degrees Celsius and the number of times a cricket chirps per minute.
   a) Find the value of $f(10)$, $f(15)$, $f(20)$, and $f(40)$.
   b) How would you interpret $f(0)$?
   c) Write an expression for $f(c)$, where c is a number that makes sense in this context.

3. How would you write in functional notation the relationship between
   a) the area of a circle and its diameter?
   b) the volume of a sphere and its radius?
   c) temperature in degrees Fahrenheit and in degrees Celsius?

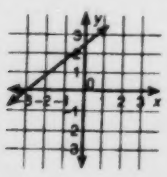
4. A scientist studying the growth of a population of small marsupials used N to stand for the number of animals in the population and t for the number of months since he began the study.
   a) He began by writing $N = f(t)$. Explain what this means.
   b) The scientist found that a suitable model for the growth of the population is given by the formula $f(t) = 50 + 4t + 0.2t^2$. Find the values of $f(0)$, $f(1)$, and $f(-1)$ and explain how you would interpret them.

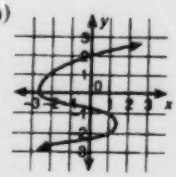
5. a) If $f(x) = x^2 + 1$, write $f(0)$, $f(1)$, and $f(-1)$.
   b) For $g(z) = \dfrac{1}{z-1}$, find $g(0)$, $g(2)$, and $g(10)$.

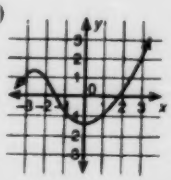
6. Determine whether each of the following is a function:
   a) $\{(1, 0), (1, 1), (1, 2)\}$
   b) $\{(0, -1), (1, -2), (2, -3), (3, -4)\}$
   c) $\{-2, -3, -4, -5, -6\}$
   d) $\{(0, 0)\}$
   e) $y^2 = 4x$
   f) $2x - 3y = 4$
   g) $y^2 = 6x$
   h) $y = 2x^2$

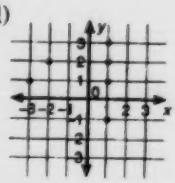
Exercise 2 (continued)

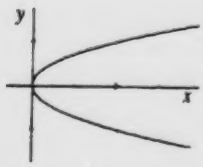
7. Which of the graphs below represent functions?

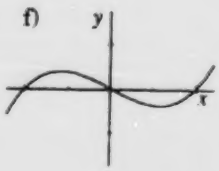
a) 

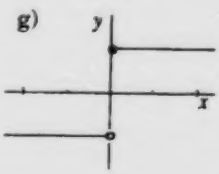
b) 

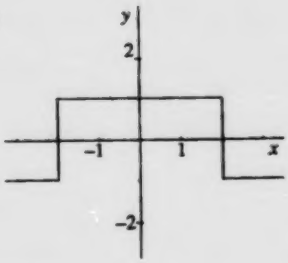
c) 

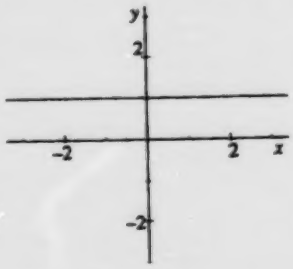
d) 

e) 

f) 

g) 

h) 

i) 

8. Investigate the domain and range of the following functions with the aid of graphing technology.

a) $f(x) = x - 2$

b) $g(x) = 9 - x^2$

c) $f(x) = \sqrt{9 - x^2}$

d) $g(x) = x^2 - 1$

e) $f(x) = \frac{1}{x}$

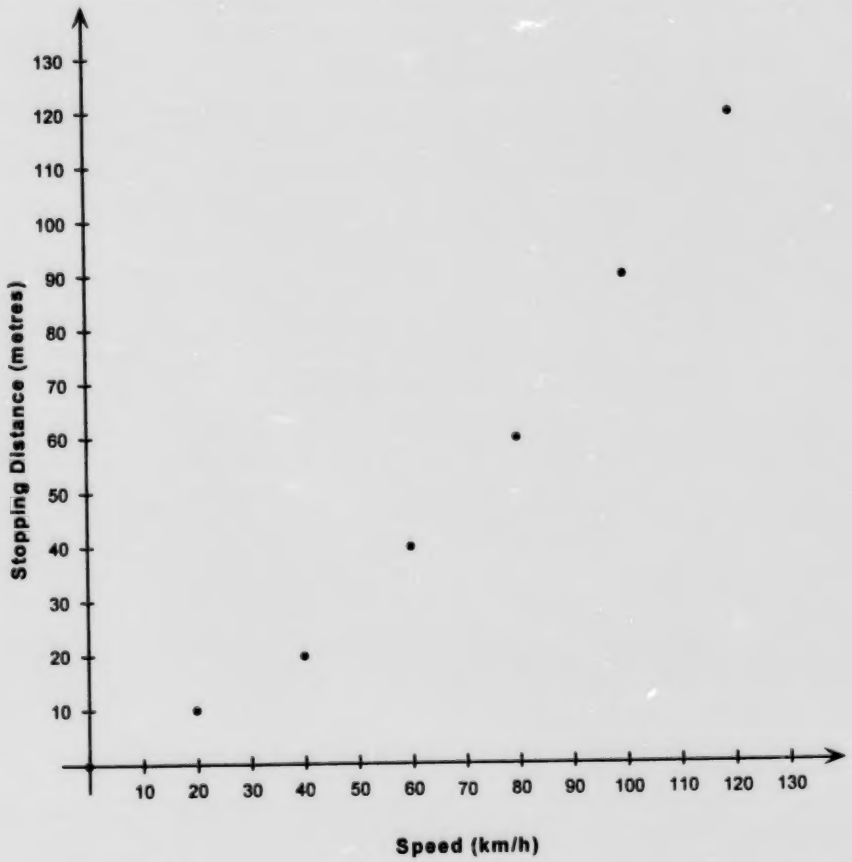
f) $g(x) = \frac{1}{x - 1}$

Unit F
Relations and Functions
Solutions

Exercise 1 Solutions

1. a) Speed is the independent variable, and stopping distance is the dependent variable.

   b)



   c) It *does* make sense to join the points, because it makes sense to talk about stopping distances for speeds between those given in the table.

   d) From the graph, we can estimate that at 50 km/h the stopping distance is approximately 28 m and at 90 km/h the stopping distance is about 73 m. These answers would be fairly accurate, provided the graph is drawn with care.

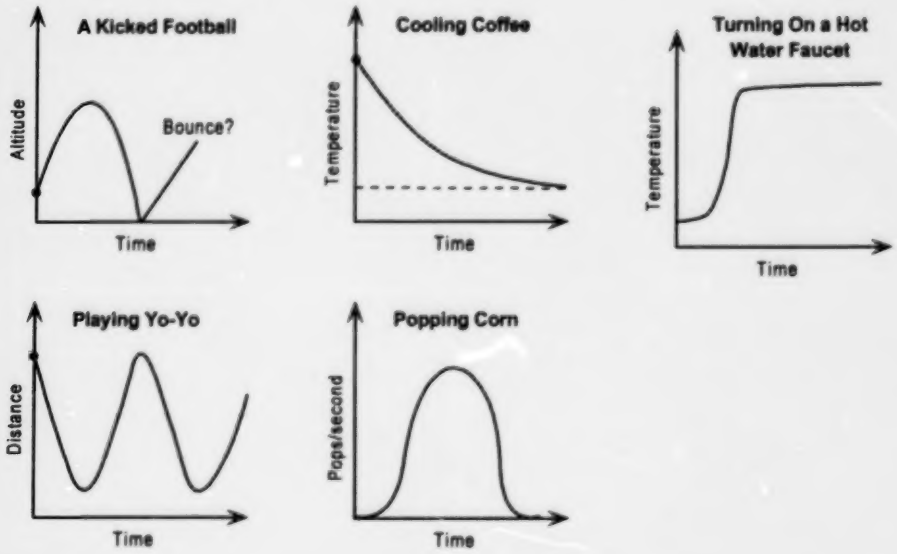
   It is hard to estimate a stopping distance for 150 km/h because this is too far beyond the last known point on the graph. A rough guess might be 170 to 180 m.

2. a) (ii)
   b) (ii)
   c) (i)

Exercise 1 Solutions (continued)

3. a) A child swinging on a swing.
   b) A child climbs up a slide and then slides down.

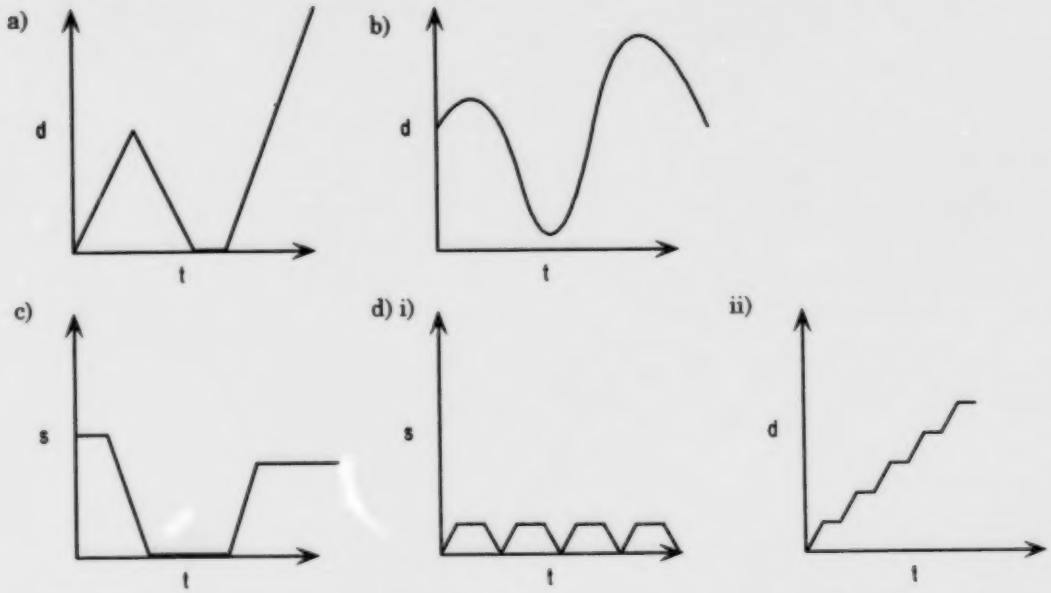
4.



5. a) (iv)
   b) Answers will vary.

Exercise 1 Solutions (continued)

6. Answers may vary. (Some possible graphs are shown below.)



7. a) The tank holds at least 35 L. It might hold more, but there is no way of telling how much more.
   b) The driver filled up twice, at 100 and 200 km from home.
   c) Most gas was bought at the first stop (25 L). Only 20 L was bought at the second stop.
   d) The tank held 20 L at the beginning and end of the journey, so the amount used is the amount bought, 45 L.
   e) The car would have run out of gas about 140 km from home.
   f) Since town driving uses more gas, the graph will be steeper at places where the driving was through a town. It is steeper at the beginning, so the person lived in a town.
   g) The town would be about 200 km from home. The rest of the trip was in the country.
   h) The first gas station was in the country, the second in the centre of a town.
   i) If there had been no stop at the second gas station, the tank would have been empty 300 km from home.

8. a) Yes. Since the chairlift moves at a rate of 400 feet per minute and it must go 2000 feet to reach the top, in general, a skier is on it for five minutes. Looking at the graph that shows speed as a function of time, notice that speed is at zero after three minutes, indicating that the skier is sitting on the chairlift.
   b) Between the third and fourth minute and the ninth and tenth minute, the distance graph is horizontal, indicating that the distance function is constant during the times. The fact that the distance does not change here reveals that the speed is zero as distance travelled equals speed multiplied by the time elapsed.

Exercise 1 Solutions (continued)

c) Approximately 4.55 mph

d) 8.2 mph

e) Possible answer: The skier probably paused to take a look at the view, because if he or she had fallen, the speed would have dropped more abruptly.

f) Answers may vary.

9. a) Illinois

b) Washington

c) Florida

d) California

Chicago, Illinois has the largest range in temperature. San Diego, California has the smallest range in temperature.

10. a) It snowed mainly on Monday and on Thursday.

b) The greatest amount fell on Monday.

c) The total depth of new snow that fell this week was approximately 47 cm.

d) A lot of snow melted on Wednesday, so that may have been the day it rained.

e) About 15 cm melted that day.

f) If the snow had continued to melt at the same rate for one more day, there would have been 10 cm left at the end of the day.

Exercise 2 Solutions

1. a) Before using functional notation, define the variables involved and decide what symbols to use for them. The first variable is a measure of how hungry a person is. There is no standard way of measuring this, and no recognized unit of hunger. Discuss with other students how you could measure hunger. Perhaps it could be measured on a scale from 1 to 10.

   Let H be a variable that measures my hunger, and let t be the time in hours since my last meal. Then the statement that my hunger depends on the length of time since my last meal is expressed in functional notation by writing $H = f(t)$.

   b) Let N be the number of people in the city suffering from asthma, and p the concentration of pollen grains in the atmosphere. Then $N = f(p)$.

2. a) $f(10) = 160$, $f(15) = 196$, $f(20) = 232$, $f(40) = 376$

   b) The function $f(0)$ represents the number of times a cricket would chirp per minute at a temperature of 0^0C.

   c) $f(c) = 7.2c + 88$

3. a) If d represents the diameter of a circle and A its area, then $A = f(d) = \frac{1}{4}\pi d^2$.

   b) If r is the radius of a sphere and V its volume, then $f(r) = \frac{4}{3}\pi r^3$.

   c) If F represents temperature in degrees Fahrenheit and C represents temperature in degrees Celsius, then $F = f(C) = 1.8C + 32$ or $C = g(F) = \frac{F - 32}{1.8}$.

4. a) $N = f(t)$ means that the number of animals in the population depends on time.

   b) $f(0) = 50$, $f(1) = 54.2$, and $f(-1) = 46.2$. The population size was 50 when the study began. Since the population has to be a whole number of animals, we should round the answers to the nearest integer. We can interpret $f(1) = 54.2$ to mean that one month after the study began there were approximately 54 animals in the population. Similarly, we can interpret $f(-1) = 46.2$ to mean that a month *before* the study began there were 46 animals. However, this goes outside the domain in which observations were made, so it may be a less reliable estimate.

5. a) $f(0) = 1$, $f(1) = 2$, $f(-1) = 2$

   b) $g(0) = 1$, $g(2) = 1$, $g(10) = \frac{1}{9}$

6. a) No b) Yes c) No d) Yes e) No f) Yes g) No h) Yes

7. a, c, f, g, i

Exercise 2 Solutions (continued)

8. a) Domain $= \{x \mid x \in \Re\}$
   Range $= \{y \mid y \in \Re\}$

   b) Domain $= \{x \mid x \in \Re\}$
   Range $= \{y \mid y \leq 9, y \in \Re\}$

   c) Domain $= \{x \mid -3 \leq x \leq 3, x \in \Re\}$
   Range $= \{y \mid 0 \leq y \leq 3, y \in \Re\}$

   d) Domain $= \{x \mid x \in \Re\}$
   Range $= \{y \mid y \geq -1, y \in \Re\}$

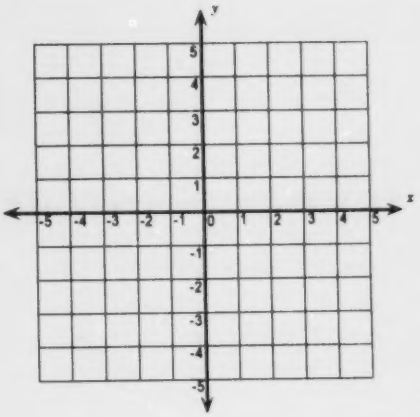
   e) Domain $= \{x \mid x \neq 0, x \in \Re\}$
   Range $= \{y \mid y \neq 0, y \in \Re\}$

   f) Domain $= \{x \mid x \neq 1, x \in \Re\}$
   Range $= \{y \mid y \neq 0, y \in \Re\}$

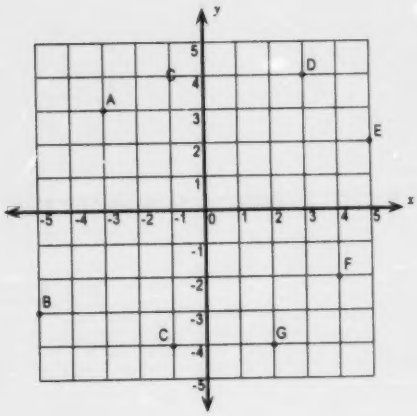
Unit G
Coordinate Geometry

Exercise 1

1. Plot the following points on the Cartesian plane:
   A (–3, 3) B(–2, –3)
   C (3, –2) D(2, 3)
   E(–2, 5)



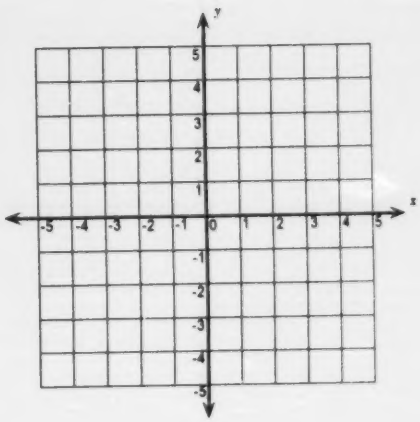
2. What are the coordinates of each point on the graph?



3. Plot the following points:
   L (–4, –3) M(–2, 4)
   N(0, –3) O(2, 4)
   P(4, –3)

   Join L to M, M to N, N to O, and O to P.

   What figure have you drawn?



Exercise 1 (continued)

4. Rectangle ABCD has vertices at A(2, 6), B(8, 6), C(8, 2), and D(2, 2). Draw rectangle ABCD on a coordinate plane and then determine its area.

5. A parallelogram has its vertices in each of the quadrants. Only three of them are given: A(–8, 2), B(2, 2), and C(7, –1). Name the coordinates of the missing vertex. After determining the missing coordinate of the parallelogram, determine the area of the parallelogram.

6. An ordered pair describes the coordinates of a point. How can you tell when a point is
   a) on the x-axis?
   b) on the y-axis?
   c) not on any axis?

7. Three vertices of a square are A(–2, –6), B(5, 1), and C(–2, 1). Find the coordinates of the fourth vertex.

8. Use graph paper to draw a coordinate plane. Plot a point that satisfies each of the following conditions:
   a) The x-coordinate is 4 greater than the y-coordinate.
   b) The x-coordinate and the y-coordinate are the same.
   c) The x-coordinate is the inverse of the y-coordinate.
   d) The product of the coordinates is –28.
   e) The x-coordinate is the square of the y-coordinate.
   f) The sum of the coordinates is 14.

9. A set of conditions is given for four points. Name the quadrant in which each point lies.
   a) $x > 0, y > 0$
   b) $x < 0, y = 2$
   c) $x > 0, y < 0$
   d) $y > 0, x = 4$

10. Plot the points A(2, 2), B(0, 2), C(–2, 0), and D(2, 0). Join the points in the given order. What type of figure is formed? Calculate the area of this figure.

11. Plot the points D(5, 1) and E(3, –3) on a coordinate plane. Find the coordinates of a third point, F, so that triangle DEF is an isosceles triangle and DE = DF. Determine at least three possible solutions.

12. Point A is (–5, –2). Point B (–2, 0) is $\frac{1}{3}$ of the way from point A to point C. What are the coordinates of point C?

Exercise 2

An important skill when working with coordinate geometry is the ability to calculate the distance between two points on a coordinate plane.

Use the Pythagorean theorem in the first part of this assignment to solve the questions that follow:

1. Complete the following sequence to understand how to use the Pythagorean theorem.
   a) Using graph paper, plot points A(4, 5) and B(1, 1).
   b) Draw the vertical line that contains point A and the horizontal line that contains B. Label their intersection C. What kind of triangle is Δ ABC?
   c) Count the number of units for line AC and line BC.
   d) Use the Pythagorean theorem to find AB.

2. Find the distance between points R(–5, 6) and S(3, –5).

3. Calculate the distance between A(1, 2) and B(4, 6).

4. Line segment CD has coordinates C(4, 8) and D(–2, 2).
   Line segment XY has coordinates X(5, –2) and Y(–2, 5).
   Which segment is longer?

5. Three vertices of a rectangle ABCD are given: A(–4, 3), B(6, 3), and C(6, 7).
   a) Determine the coordinates of point D.
   b) Find the lengths of the sides.
   c) Determine the length of the diagonals.

6. Plot the following information on a coordinate plane. Assume that you are starting at the origin (0, 0).

   To get to a hospital from the middle of a nearby village, you can drive an ambulance 8 km east, turn and go 4 km south, and then go 1 km west. As you can see, this means an ambulance must drive 13 km to get to the hospital. Determine how much shorter it would be for the helicopter rescue unit to go to the hospital.

7. Which point is further from the origin: P(7, 1) or Q (6, 4)?

In the second part of this exercise, use the distance formula to solve the following questions:

8. Find the distance between A(7, 8) and B(–5, 3).

9. Find the distance between X(–2, 5) and Y(3, 7).

10. Plot the points A(–1, 1), B(3, 2), and C(3, –1) on a coordinate plane. Join the points to form Δ ABC. Using the diagram and the distance formula, determine the perimeter of Δ ABC correct to one decimal place.

Exercise 2 (continued)

11. A seaport is located at P(–9, 4), a fishing boat is located at B(3, 7), and a yacht is located at Y(7, –5). If each unit represents a sea mile (knot), calculate to two decimal places the distance between:
    a) the boat and the yacht
    b) the port and the boat
    c) the port and the yacht

12. The vertices of a triangle are D(5, –1), E(10, –12), and F(–1, –7). Find the area of Δ DEF. (Hint: Draw a diagram to assist you to find the height of the triangle.)

13. A triangle has vertices A(7, 7), B(–8, –13), and C(–8, 7). Find the perimeter of Δ ABC. Classify the triangle as scalene, isosceles, or equilateral.

14. Three vertices of a rectangle are D(–5, –2), E(7, 2), and F(9, –4).
    a) Find the coordinates of the fourth vertex G.
    b) Find the lengths of the sides of the rectangle.
    c) What is the area of the rectangle?
    d) Find the length of the diagonal of the rectangle.

15. A triangle has vertices X(3, 4), Y(4, –1), and Z(–1, –2). Explain why this triangle is isosceles.

16. A triangle has vertices A(5, 6), B(1, 3), and C(4, –1). Prove that this is a right triangle. (Hint: Use the converse of the Pythagorean theorem.)

17. The distance from point A to (2, 8) is 17.
    a) Show that A could be (10, 23).
    b) Name five other possible locations of point A. (Hint: Draw a picture.)

Exercise 3

1. Find the coordinates of the midpoint of the line segment with these end points:
   a) A(1, 6) B(9, 6)
   b) D(–3, 3) E(–9, 3)
   c) G(6, –1) H(6, –7)
   d) M(–7, 4) N(–7, –4)
   e) P(4, 4) Q(8, 4)
   f) R(3, 8) S(3, 4)

2. Find the midpoint of each side of triangle ZYZ whose vertices are X(12, 4), Y(–6, 2), and Z(–4, –2).

3. One end point of line segment DE is D(–2, 4). If the coordinates of the midpoint are (–1, 7), find the coordinates of E.

4. A right triangle has vertices (–2, 8), (6, 4), and (4, 0).
   a) Draw the triangle on a grid and find the coordinates of the midpoint, M, of the hypotenuse.
   b) Show that M is equidistant from all three vertices.

5. The endpoints of the diameter of a circle are (–4, –2) and (6, 4).
   a) Find the coordinates of the centre of the circle.
   b) Find the radius of the circle to one decimal place.
   c) Find the circumference and area of the circle to one decimal place.

6. The midpoint of PQ is the origin. P has coordinates (4, 5). What are the coordinates of Q?

7. The vertices of a parallelogram are A(2, 1), B(7, 2), C(8, 5), and D(3, 4). Find the midpoint of each diagonal.

8. The coordinates (–1, 6) represent the midpoint of a line segment. If (2, 8) is one endpoint, what is the other endpoint?

9. A rectangle has vertices A(1, 3), B(6, 5), C(8, 0), and D(3, –2).
   a) Determine the length of the two diagonals AC and BD.
   b) Find the midpoints of the diagonals AC and BD.
   c) What do you notice about the midpoints in part (a)?

10. One endpoint of a segment AB is A(–3, 8). If the coordinates of the midpoint are (2, 2), find the coordinates of B.

Exercise 4

1. Locate the points A(2, 3), B(6, 3), C(2, 1), and D(2, –1) on a graph and determine the slopes of each of these line segments.

   a) $\overline{AC}$

   b) $\overline{CA}$

   c) $\overline{AB}$

   d) $\overline{DA}$

   e) $\overline{DC}$

2. Graph these line segments and find their slopes using slope = $\frac{\text{rise}}{\text{run}}$

   a) A(–2, 7) B(6, –4)

   b) C(3, –5) D(8, 10)

   c) E(1, 6) F(5, –4)

   d) G(–3, 7) H(–3, –7)

   e) J(–4, –3) K(8, 5)

   f) L(2, –7) M(7, –7)

3. A section of roller-coaster track falls 25 m in a horizontal distance of 15 m. What is the slope of this section of track?

4. A new overpass is to be connected to an access road 250 m away by a road of uniform gradient. If the overpass is 8 m above the access road, what will be the slope of the connecting road?

5. On a grid, draw a line through

   a) (–3, 5) with slope $\frac{-4}{7}$

   b) (–2, – 6) with slope $\frac{3}{4}$

   c) (5, – 4) with slope $\frac{-8}{3}$

   d) (6, 4) with slope undefined

   e) (–3, 1) with slope $\frac{7}{3}$

   f) (6, 4) with slope 0

6. A contractor is installing curb ramps on all its sidewalks. The ramps are 12 cm high and the slope of the ramp is to be 0.4. How far from the curb must each ramp start?

7. The slope of a playground slide is 1.4. If the run is 1.5 m and the steps on the ladder are 30 cm apart, how many steps are on the ladder?

Exercise 4 (continued)

8. You are walking up a ramp with a slope of 0.2. If you moved 10 m in a horizontal direction, how many metres have you moved up?

9. Two points on a line are (4, 3) and (6, 4). Determine the slope of the line. Then, determine the coordinates of two points on the line, one of which is in a quadrant other than the quadrant in which the original two points are located.

Algebraically, the slope formula is given as: $\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$

Use this formula to assist you in the questions that follow:

10. Find the slope of the line segment joining P(1, 2) and Q(4, 5).

11. Calculate the slope of the line through A(1, 4) and B(3, 4).

12. Find the slope of the line through C(1, 2) and D(1, 4).

13. A line passes through the points (4, 3) and $(-2, k)$. The slope of the line is 0.5. Find the value of k.

14. A line passes through the points $(2, k)$ and $(-3, 2k)$. The slope of the line is –0.5. Find the value of k.

15. A line passes through the points (2, –3) and $(4, y)$. Find y if the slope of the line is $\frac{2}{3}$.

16. Three nonvertical lines, each containing the origin, have slopes 5, –2, and n. What is the y-coordinate of the point on each line if the x-axis coordinate is 1?

17. Use the points A(–3, –2) and B(7, 4) to determine the following:
    a) slope of AB
    b) value of k if the point $C(2, k)$ lies on AB

18. Determine whether or not the quadrilateral A(0, –6), B(2, –1), C(–1, 5), D(–3, 0) is a parallelogram.

19. Plot the points A(1, 3) and B(3, 7).
    a) Determine the slope of AB.
    b) Using the slope from part (a), determine the missing coordinate for each of the following points that lie on the same line as A and B.
       - C(5, ?)
       - D(–1, ?)
       - E(?, –5)
       - F(9, ?)

Exercise 5

1. Graph the following on one pair of axes:

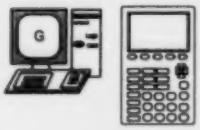
   a) $y = 3x$ b) $y = 2x$

   c) $y = x$ d) $y = \frac{1}{2}x$

   e) $y = -3x$ f) $y = \frac{-1}{2}x$

   Describe how the graph of $y = mx$ changes as m changes.

2. Graph the following on one pair of axes:



   a) $y = x + 4$ b) $y = x + 2$

   c) $y = x$ d) $y = x - 2$

   e) $y = x - 4$

   Describe how the graph of $y = mx + b$ changes as b changes, while m remains constant.

3. Each equation defines a line. State the slope and the y-intercept in each case.

   a) $y = \frac{3}{4}x - 3$ b) $y = 4x + 5$

   c) $y = \frac{-4}{3}x - 2$ d) $y = 3x$

   e) $y = \frac{-2}{3}x$ f) $y = 5 - 2x$

4. State the slope and the y-intercept of the following lines.

   a) $3x - 2y = 6$ b) $5x - 2y + 10 = 0$

   c) $2x + y - 3 = 0$ d) $3x + 5y + 20 = 0$

   e) $x + 2y - 5 = 0$ f) $4x - 7y + 15 = 0$

5. The equation of a line is in the form $y = 3x + b$. In each case, find b if the line passes through the given point.

   a) (3, 4)

   b) (–2, –3)

   c) (4, 1)

   d) (–5, 2)

Exercise 5 (continued)

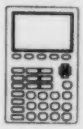
6. Graph the following using the $y = mx + b$ approach.

a) $y = 2x + 1$ b) $y = 4x + 3$

c) $y = \frac{1}{2}x - 5$ d) $2y = 6x + 4$

e) $3y = 6x - 9$ f) $y = 8x - 3$

g) $2y = x + 2$ h) $\frac{3}{2}y = 3x + 6$



7. Write equations in the form $y = mx + b$ for the following straight lines.

	Passes through	Slope
a)	(1, 7)	2
b)	(2, 1)	3
c)	(−1, 4)	$\frac{-2}{5}$
d)	(4, 0)	$\frac{3}{4}$
e)	(0, 5)	−2

8. Find the x- and y-intercepts for each line.

a) $3x - y = 6$ b) $2x = 8 + y$

c) $4x - y + 8 = 0$ d) $3y - 2x = 6$

e) $2.5x + 3.5y = 7$ f) $1.5x + 0.6y = 4.5$

9. Find the equation of the line that passes through

a) (2, 1) and (5, 7) b) (–3, 2) and (1, –10)

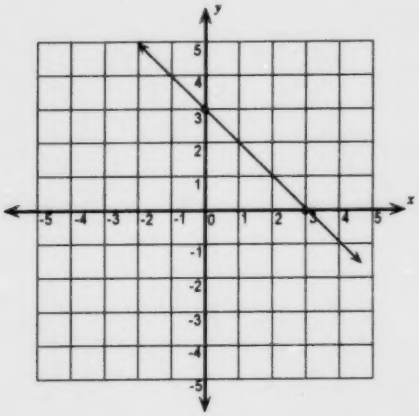
c) (5, –2) and (7, 5) d) (–1, –3) and (4, 7)

e) (4, –1) and (–2, –5) f) (–7, –12) and (–4, –4)

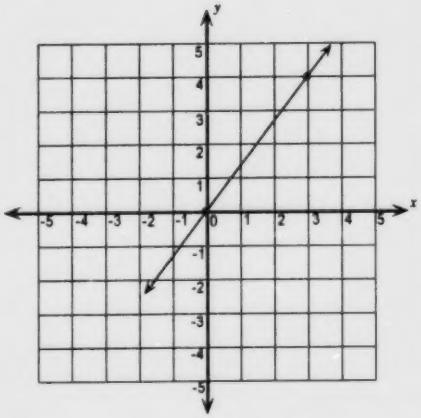
Exercise 5 (continued)

10. Write the equation of the lines shown below in the form $y = mx + b$.

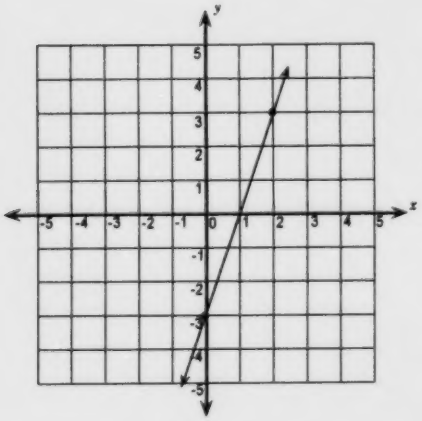
a)



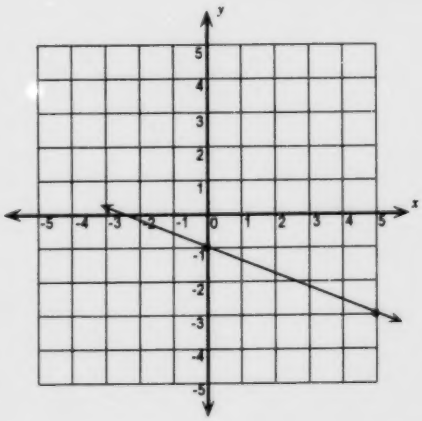
b)



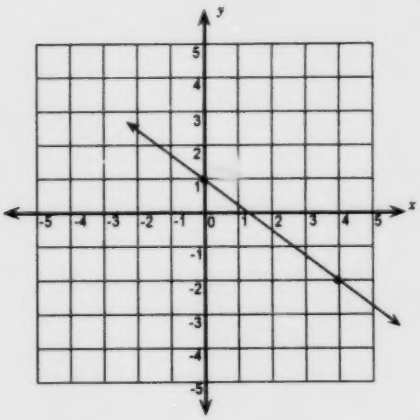
c)



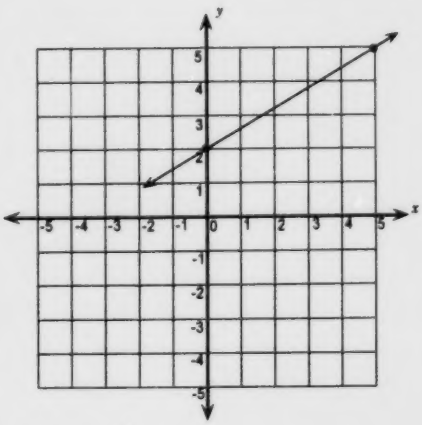
d)



e)



f)



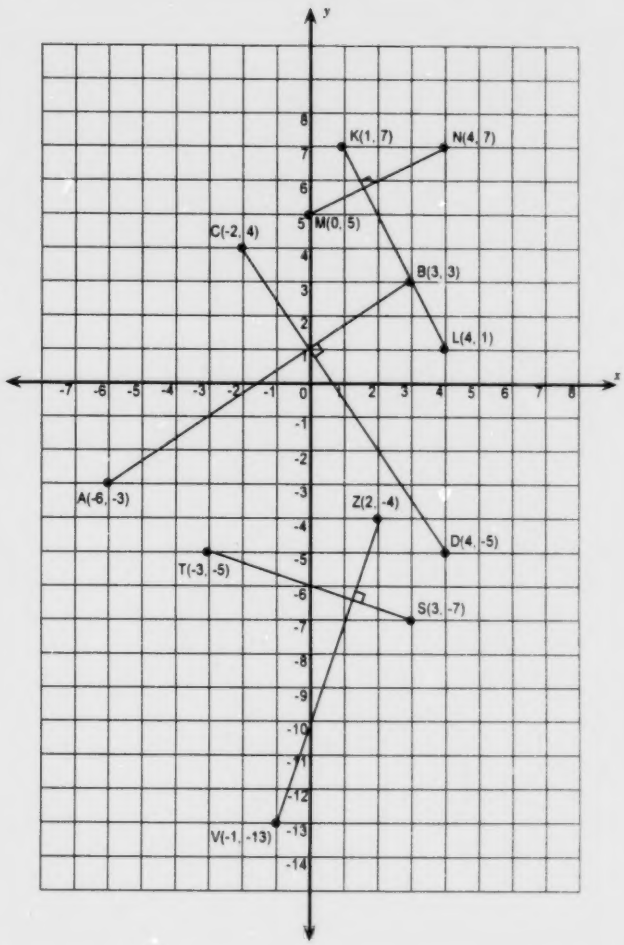
Exercise 6

1. In the diagram, pairs of intersecting line segments are perpendicular. Calculate the slope of each line segment.

   Record the results. What do you observe?

   On a piece of graph paper, draw and label the axes. Draw any two perpendicular line segments. Calculate the slope of each line segment. What do you observe?

   Write a rule for finding the slope of a line perpendicular to a given line.



2. Given the points A(1, 3), B(3, 7), C(–1, –6), and D(3, 2), determine if AB and CD are parallel.

3. Given the points P(2, 6), Q(–3, 4), R(2, –2), and S(–2, 8), determine how PQ and RS are related.

4. Determine whether or not quadrilateral A(0, –6), B(2, –1), C(–1, 5), and D(–3, 0) is a parallelogram.

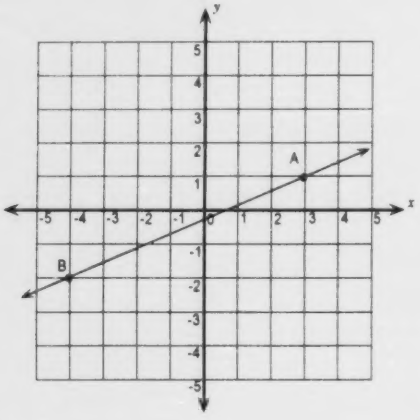
5. The slopes of two lines AB and CD are given as follows:
   - slope of AB = $m + 3$
   - slope of CD = $3m - 5$

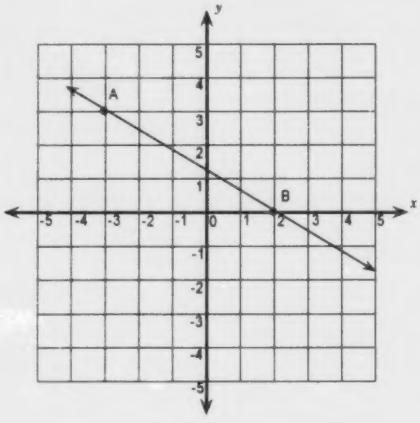
   If the lines are parallel, determine m.

Exercise 6 (continued)

6. Two line segments, with a common endpoint (0, 4), have slopes of 2 and $-\frac{1}{2}$. If the other endpoints of the line are on the x-axis, find their coordinates.

7. For each diagram shown below, what is the slope of lines that are
   a) parallel to AB?
   b) perpendicular to AB?





8. Show that the points A(–2, 1), B(–1, 4), and C(5, 2) are vertices of a right triangle.

9. Write an equation in the form $y = mx + b$ for each of the lines described below:
   a) perpendicular to $y = 4x - 1$ and having y-intercept of 3
   b) perpendicular to $y = \frac{1}{3}x + 4$ and passing through (2, 1)
   c) perpendicular to $x - 2y - 1 = 0$ and having y-intercept of –4
   d) perpendicular to $x + y = 6$ and having x-intercept of 2

10. Write an equation in the form $y = mx + b$ for each of the lines described below:
   a) passing through (4, –3) and parallel to $y = \frac{3}{4}x - 5$
   b) passing through the origin and parallel to the line through (2, –2) and (–1, 3)
   c) parallel to $y = \frac{3}{2}x - 6$ with x-intercept at –5
   d) parallel to $4x - 5y + 20 = 0$ and passing through (6, –3)

Unit G
Coordinate Geometry
Solutions

Exercise 1 Solutions

1. Students should plot A, B, C, D, and E on the Cartesian plane.
2. A(–3, 3), B(–5, –3), C(–1, –4), D(3, 4), E(5, 2), F(4, –2), G(2, –4)
3. Letter M should be formed.
4. Area = 24 square units
5. The missing coordinates are (–3, –1). The area will be 30 square units.
6. a) When a point lies on the x-axis, the second coordinate is zero.
   b) When a point lies on the y-axis, the first coordinate is zero.
   c) When a point is not on either of the axes, both coordinates of the ordered pair are non-zero.
7. D(5, –6)
8. Students' answers will vary.
9. a) I
   b) II
   c) IV
   d) I
10. A trapezoid is formed. The area is 6 square units.
11. Possibilities: (7, –3), (5, –7), (3, 5), (1, 1)
12. C(4, 4)

Exercise 2 Solutions

1. d) AB = 5 units
2. RS = 13.6 units
3. AB = 5 units
4. XY is longer
5. a) D(–4, 7)
   b) AB = CD = 10 units
   AD = BC = 4 units
   c) Diagonals AC and BD = 10.77 units
6. The helicopter could travel 8.1 km (to the nearest tenth). This means you would save 13 – 8.1 = 4.9 km in distance.
7. Q is further from the origin.
8. AB = 13 units
9. XY = 5.4 units (to the nearest tenth)
10. AB = 4.1, AC = 4.5, BC = 3, and the perimeter of Δ ABC = 11.6 units
11. a) 12.7 knots
    b) 12.4 knots
    c) 18.4 knots
12. 48 square units
13. The perimeter of Δ ABC is 60 units. This is a scalene triangle.
14. a) G(–3, –8)
    b) The rectangle is $4\sqrt{10}$ by $2\sqrt{10}$ units.
    c) The area of the rectangle is 80 units.
    d) The length of the diagonal is $10\sqrt{2}$ units.
15. Using the distance formula, you find AB = BC. By definition, triangle ABC is isosceles.
16. $AB^2 = 25$ and $BC^2 = 25$, $AC^2 = 50$. Therefore, $AB^2 + BC^2 = AC^2$ and triangle ABC is a right triangle.
17. Use the distance formula to show that the distance from A to (2, 8) is 17 units. Some sample locations include (2, 25), (2, –9), (19, 8), (–15, 8), and (10, –7).

Exercise 3 Solutions

1. a) (5, 6) b) (–6, 3) c) (6, –4)
   d) (7, 0) e) (6, 4) f) (3, 6)

2. XY (3, 3), XZ (4, 1), YZ(–5, 0)

3. E(0, 10)

4. a) M(1, 4)

5. a) (1, 1)
   b) 5.8
   c) C = 36.4 units
      A = 106.8 square units

6. (–4, –5)

7. (5, 3)

8. (–4, 4)

9. a) 7.6 units
   b) (4.5, 1.5) (4.5, 1.5)
   c) same

10. (7, –4)

Exercise 4 Solutions

1. a) undefined
   b) undefined
   c) 0
   d) undefined
   e) undefined

2. a) $\frac{-11}{8}$ b) 3
   c) $\frac{-5}{2}$ d) undefined
   e) $\frac{2}{3}$ f) 0

3. $\frac{5}{3}$

4. 0.032

5. Check graphs for location of points.

6. 30 cm

7. 7 steps

8. 2 m

9. Slope $\frac{1}{2}$. Points on the line will vary.

10. 1

11. 0

12. undefined

13. 0

14. 2.5

15. $\frac{-5}{3}$

16. 5, –2, and n respectively

17. a) $\frac{3}{5}$ b) $k = 1$

18. Slope AB = slope DC
    Slope AD = slope BC
    Since the slopes of the opposite sides are equal, the quadrilateral is a parallelogram.

19. a) Slope of AB = 2
    b) C(5, 11), D(–1, –1), E(–3, –5), F(9, 19)

Exercise 5 Solutions

1. Students should describe how the slope of the line changes.

2. Students should describe how the lines are parallel and the fact that the lines are crossing the y-axis at different locations.

3.

	slope	y-intercept
a)	$\frac{3}{4}$	-3
b)	4	5
c)	$\frac{-4}{3}$	-2
d)	3	0
e)	$\frac{-2}{3}$	0
f)	-2	5

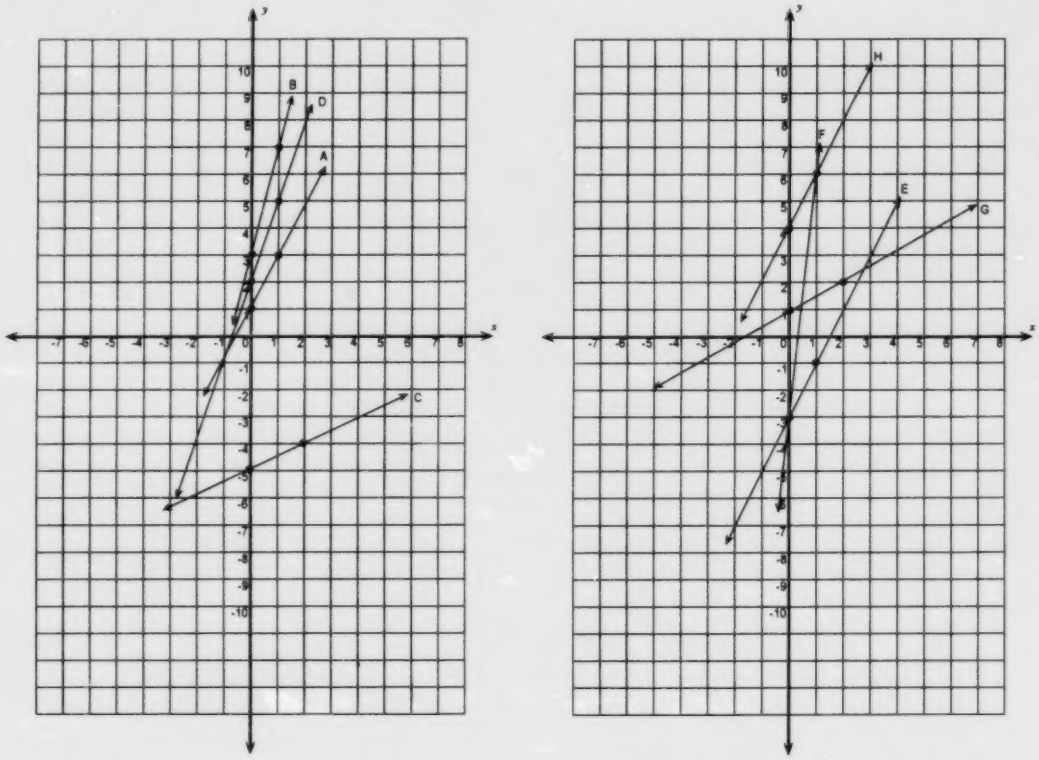
4.

	slope	y-intercept
a)	$\frac{3}{2}$	-3
b)	$\frac{5}{2}$	5
c)	-2	3
d)	$\frac{-3}{5}$	-4
e)	$\frac{-1}{2}$	2.5
f)	$\frac{4}{7}$	$\frac{15}{7}$

5. a) -5 b) 3
c) -13 d) 17

Exercise 5 Solutions (continued)

6.



7. a) $y = 2x + 5$
   b) $y = 3x - 5$
   c) $y = \frac{-2}{5}x + 3.6$
   d) $y = \frac{3}{4}x - 3$
   e) $y = -2x + 5$

8.

x-intercept	y-intercept
2	–6
4	–8
–2	8
–3	2
2.8	2
3	7.5

Exercise 5 Solutions (continued)

9. a) $y = 2x - 3$
   b) $y = -3x + 8$
   c) $y = \frac{7}{2}x - 19.5$
   d) $y = 2x - 1$
   e) $y = \frac{2}{3}x - \frac{11}{3}$
   f) $y = 8x + \frac{20}{3}$

10. a) $y = -x + 3$
    b) $y = \frac{4}{3}x$
    c) $y = 3x - 3$
    d) $y = \frac{-2}{5}x - 1$
    e) $y = \frac{-3}{4}x + 1$
    f) $y = \frac{3}{5}x + 2$

Exercise 6 Solutions

1. Students should observe that the slopes of the perpendicular line segments are negative reciprocals. They should also conclude that if two line segments have slopes that are negative reciprocals, then they are perpendicular.

2. Slope AB = 2 and slope CD = 2. Therefore, the lines are parallel.

3. Slope PQ = $\frac{2}{5}$

   Slope RS = $\frac{-10}{4} = \frac{-5}{2}$

   Therefore, the lines are perpendicular to each other.

4. Since both pairs of opposite sides are parallel, quadrilateral ABCD is a parallelogram.

5. $m = 4$

6. (–2, 0) and (9, 0)

7. 1st diagram: parallel slope $\frac{3}{7}$ perpendicular slope $\frac{-7}{3}$

   2nd diagram: parallel slope $\frac{-3}{5}$ perpendicular slope $\frac{5}{3}$

8. Slope AB = 3; Slope AC = $\frac{1}{7}$; Slope BC = $\frac{-1}{3}$

   Since AB is a negative reciprocal to slope BC, the two lines are perpendicular and you have a right triangle.

9. a) $y = \frac{1}{3}x + 4$ b) $y = -3x + 7$

   c) $y = -2x - 4$ d) $y = x - 2$

10. a) $y = \frac{3}{4}x - 6$ b) $y = \frac{-5}{3}x$

    c) $y = \frac{3}{2}x + 7.5$ d) $y = \frac{4}{5}x - 7.8$

Unit H
Measurement Technology

Exercise 1

Identifying suitable units of measure in the S.I. and imperial systems

1. Complete the table using the most reasonable units of measure for the following:

	Item	Metric	Imperial
1	Package of rice		
2	Weight of Aunt Sue		
3	Weight of an elephant		
4	Sugar for a cake		
5	Expensive French perfume		
6	Distance from Ottawa to Edmonton		
7	Bottle of soda		
8	Container of eggnog		
9	Plastic pail of ice cream		
10	Tank of gasoline		
11	Neck size of man's shirt		
12	Material for draperies		

Exercise 1 (continued)

Choosing a reasonable measure for the unit selected

2. Complete the table using a reasonable measure for each item selected.

	Item	Metric	Imperial
1	Height of a 6-year-old child		
2	Diameter of a dime		
3	Distance from Halifax to Vancouver		
4	Length of a new pencil		
5	Amount of snowfall in one night		
6	Width of videotape		
7	Weight of average adult male		
8	Weight of large apple		
9	Weight of a tablespoon of salt		
10	Weight of a vitamin C tablet		
11	Weight of a dinner plate		
12	Weight of a bag of dog food		
13	Volume of the fuel tank of a car		
14	Amount of fluid a person drinks daily		

Exercise 2

Conversions — Metric Units

1. Change the following units of metric measure as indicated.

   a) 6 m = _____ cm

   b) 6.5 cm = _____ mm

   c) 19 L = _____ mL

   d) 3 m = _____ mm

   e) 64 km = _____ m

   f) 0.35 km = _____ m

   g) 853 mL = _____ L

   h) 12.4 kg = _____ g

   i) 3423 mm = _____ m

   j) 2250 g = _____ kg

   k) 175 cm = _____ m

2. Change the following area and volume measures as indicated.

   a) $1\ m^2 =$ _____ cm^2

   b) $1\ m^3 =$ _____ cm^3

   c) $1\ m^2 =$ _____ mm^2

   d) $1\ m^3 =$ _____ mm^3

   e) $3.5\ m^2 =$ _____ m^2

   f) $2.8\ m^3 =$ _____ cm^3

   g) $12\ 560\ cm^2 =$ _____ m^2

   h) $1850\ mm^2 =$ _____ cm^2

   i) $135\ 000\ mm^3 =$ _____ m^3

3. Add or subtract these measures:

   a) 3 m + 45 cm =

   b) 525 g + 3.5 kg =

   c) 14.5 cm – 15 mm =

   d) 2.5 L + 125 mL =

   e) 22.4 km – 225 m =

   f) 2.15 m + 14 mm + 25.8 cm =

Exercise 2 (continued)

Conversions — Imperial Units

4. Change the units as indicated.

   a) 5.25 ft. = _____ in.

   b) 2 yd. 1 ft. 3 in. = _____ in.

   c) 3.5 mi. = _____ ft.

   d) 75 in. = _____ ft. (decimal)

   e) 4 in. = _____ ft. (fraction)

   f) 72 in. = _____ yd.

5. Do the operations as indicated. Write your answers in feet and inches.

   a) (2 ft. 9 in.) + (7 ft. 8 in.) =

   b) (4 yd. 8 in.) – (2 yd. 1 ft.) =

   c) 4 • (2 ft. 7 in.) =

   d) 3(1 yd. 2 ft. 8 in.) =

6. Do the following operations. Similar operations will occur when determining measurements with a Vernier caliper.

   a) $2\frac{1}{8}$ in. + $\frac{3}{16}$ in. =

   b) $1\frac{5}{8}$ in. + $\frac{3}{128}$ in =

   c) $\frac{7}{8}$ in. + $\frac{3}{16}$ in. =

   d) $3\frac{3}{16}$ in. + $\frac{7}{8}$ in. =

   e) $\frac{1}{8}$ of $\frac{3}{16}$ in. =

   f) $\frac{7}{8}$ of $\frac{5}{16}$ in. =

   g) $\frac{5}{8}$ of $\frac{5}{16}$ in. =

   h) $\frac{3}{8}$ of $\frac{1}{16}$ in + $2\frac{3}{4}$ in. =

   i) $\frac{5}{8}$ of $\frac{1}{16}$ in. + $3\frac{3}{8}$ in. =

   j) $\frac{1}{8}$ of $\frac{1}{16}$ in. + $1\frac{1}{2}$ in. =

Exercise 3

Elements in technology and industry/construction

Metric units of volume

1. Name two things that might be appropriately measured in
   a) cubic metres
   b) cubic centimetres
2. Explain why 1 cm = 10 mm, but 1 cm^3 = 1000 mm^3.
3. How many cubic millimetres are there in
   a) 1 cubic centimetre?
   b) 1 cubic metre?
4. Change to cubic millimetres.
   a) 4.5 cm^3
   b) 13 m^3
5. Change to cubic centimetres.
   a) 13 m^3
   b) 5670 mm^3
6. Change to cubic metres.
   a) 44 000 cm^3
   b) 22 400 000 cm^3
7. A building contractor excavates a basement that measures 10 m by 13 m by 2.3 m. A truck with a 2.3-m by 4.5-m by 1.4-m box is used to haul the soil. Soil has a density of 1 320 kg/m^3.
   a) How many cubic metres of soil are removed from the basement?
   b) How many truck loads are required to haul the dirt away?
   c) What is the total mass of soil and truck if the truck unloaded has a mass of 1 328 kg?

Exercise 3 (continued)

Metric volumes, capacities, and mass

8. Change the following units as indicated:
   a) 42 millilitres to cubic centimetres
   b) 13 litres to cubic centimetres
   c) 542 cubic centimetres to millilitres
   d) 5 870 cubic centimetres to litres

9. How much space, in cm^3, is occupied by the capacities shown?
   a) 32 mL
   b) 0.45 L

10. What capacity will match the following volumes? Write your answer in millilitres.
   a) 23 cm^3
   b) 1 100 mm^3
   c) 0.027 m^3

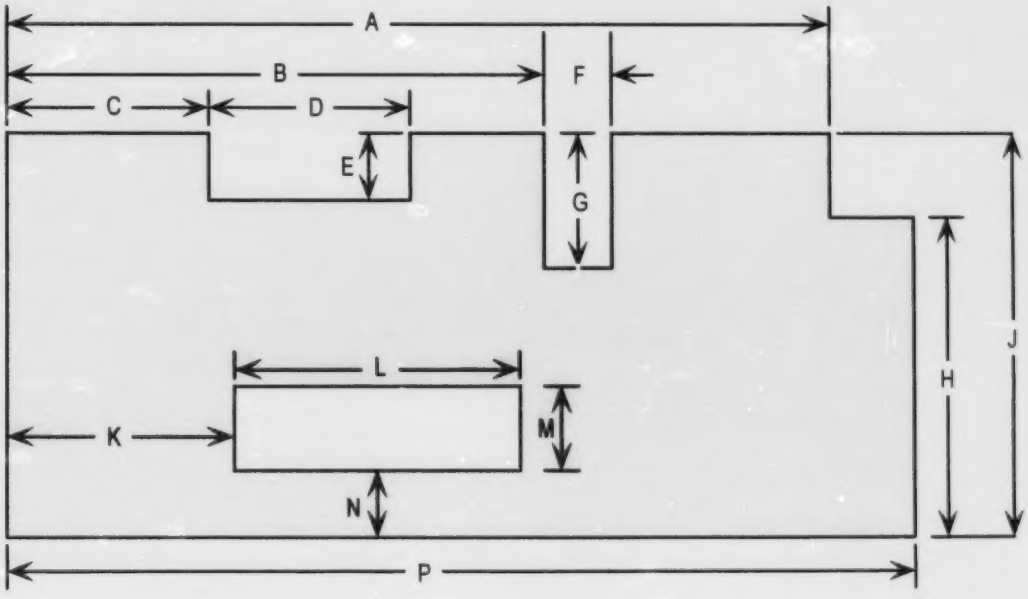
11. Write the capacity in litres for each volume given.
   a) 2.3 m^3
   b) 7 500 cm^3
   c) 25 000 mm^3

12. Solve the problems.
   a) A car engine holds about 4.5 L of oil. Rewrite this in cm^3. If oil has a mass of 0.85 g/L, what is the mass of the oil? Write your answer in kilograms.
   b) The inside dimensions of an aquarium are 62 cm long by 30 cm wide by 38 cm deep. What is the mass of water in the aquarium if it is filled to a depth of 36 cm? Write your answer in the most suitable units.

Note: Be sure to save all spreadsheet work on a disk for future reference.

Exercise 4

Measuring with a metric ruler



Use the above diagram to take the following measurements:

1. a) Use a metric ruler to measure the lettered dimensions in the above diagram. Round your answers to the nearest millimetre.

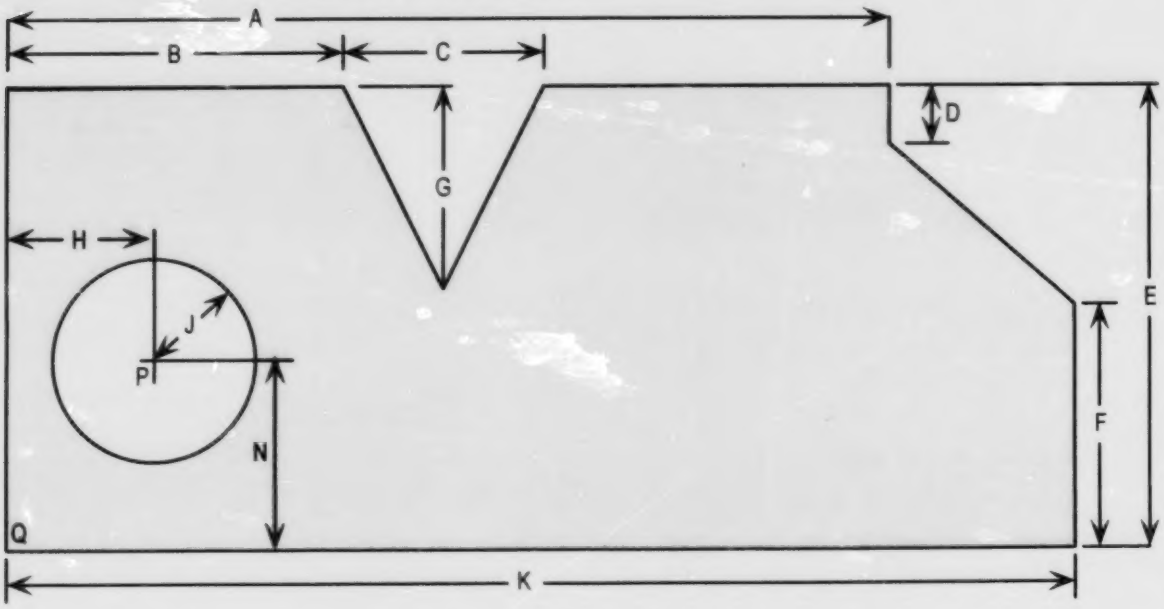
   b) Find the area of the shaded surface. Write your answer in square centimetres rounded to two decimal places.

   c) Determine the volume of the above shape when it is cut from material 6 mm thick. Write your answer, first, in cubic millimetres rounded to two decimal places. Then, write the volume in cubic centimetres rounded to three decimal places.

   d) Find the mass of the above shape if it is made out of brass with a density of 8.9 g/cm^3. Write your answer, first, in grams to one decimal place, and then in kilograms to three decimal places.

Exercise 4 (continued)

Measurements with an imperial scale (ruler)



1. a) Measure the lettered dimensions shown. Write your answers correct to the nearest $\frac{1}{16}$ of an inch.

   b) Find the area of the triangular piece that is cut out of the top.

   c) Find the area of the disk.

   d) Find the area of the shaded surface. Write your answer in square inches rounded to two decimal places.

   e) Calculate the distance PQ if P is the centre of the circle, and Q is the lower left point.

   f) The shape is to be cut from a rectangular piece of plastic that has the outside dimension of K by E. Obviously, some material will be recycled. Calculate the area of material for that use. Further, calculate what percentage of material was recycled if 100 shapes were cut, each from sheets measuring K by E.

   g) The thickness of the plastic used is $1\frac{5}{16}$ inches. Calculate the volume. Write your answer as cubic inches rounded to two decimal places.

   h) Repeat questions (a) to (e) for a piece of plastic 1.4 inches thick, using a scale with $\frac{1}{10}$ of an inch increments.

   i) Which scale do you find more convenient to use for this project — a $\frac{1}{16}$ or $\frac{1}{10}$ scale? Explain why you think one scale is better than the other.

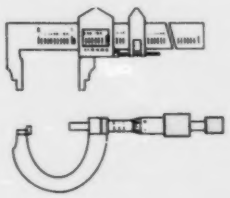
   j) Write the advantages and disadvantages of using either the metric or imperial system of measure.

Exercise 5

Using a micrometer to measure distance

1. a) Measure a variety of items using the micrometer. Record your data in a chart as shown below. Use the highest degree of accuracy possible with the micrometer that you are using.

Item	Measure
Paper	
Hair	
Pencil lead (0.5 mm)	
Bolt head	



Other items that could be measured might include pipes, rods, pencil, nuts, bolts, washers, wires (a variety of gauges), or nails.

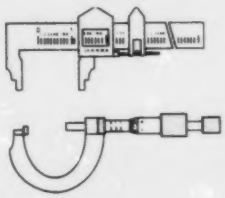
b) Which instrument (ruler, caliper, or micrometer) would be most suited to measuring

i) the length of a bolt to the nearest $\frac{1}{8}$ of an inch?

ii) the width of the head of a bolt to the nearest 0.01 of an inch?

iii) the diameter of the shank of the bolt to the nearest 0.001 of an inch?

iv) the diameter of the plastic or copper water pipe at the hardware store?



2. Using various items such as drill bits and different wire sizes, compare standard sizes with micrometer readings of various objects.

a) Piston diameter

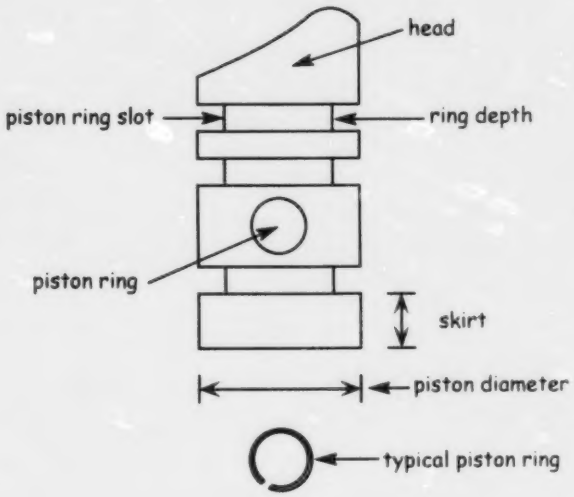
i) above piston pin

ii) at skirt

b) Thickness of piston rings

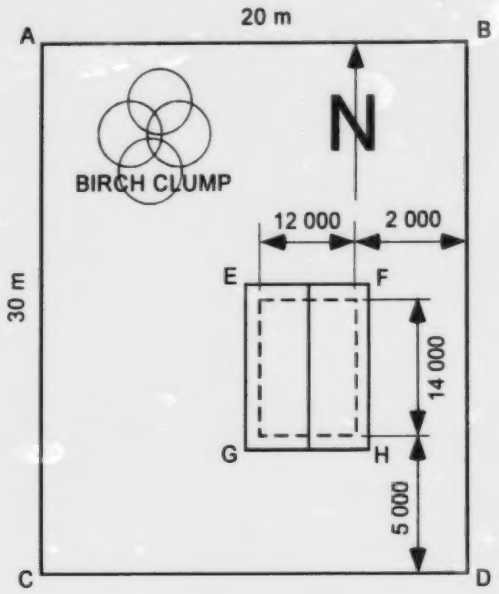
i) above piston pin

ii) at skirt



Exercise 6

Land site work and measurement



Measurements within this site plan are in millimetres.

Not to Scale

The following questions apply to the rectangular site plan above:

1. a) In what corner of the lot is the birch clump located?

   b) How far north of the south property line is the residence located?

   c) How far west of the east property line is the residence located?

   d) An electrical conduit is to be placed underground from point C to point G on the residence. How long will that conduit be?

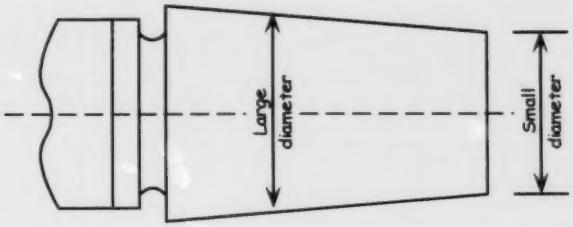
   e) Given that pegs are placed at A, B, C, and D, describe how the resident engineer could verify that the lot is rectangular.

Exercise 7

Measurement Problems

1. Find the difference in diameters (taper) of the following plugs: A, B, C, D, and E.

Plug	Diameter	
	Large End	**Small End**
A	1.500"	1.250"
B	4.0824"	3.748"
C	5.4223"	4.576"
D	11.2654"	9.4237"
E	6.0672"	4.8923"



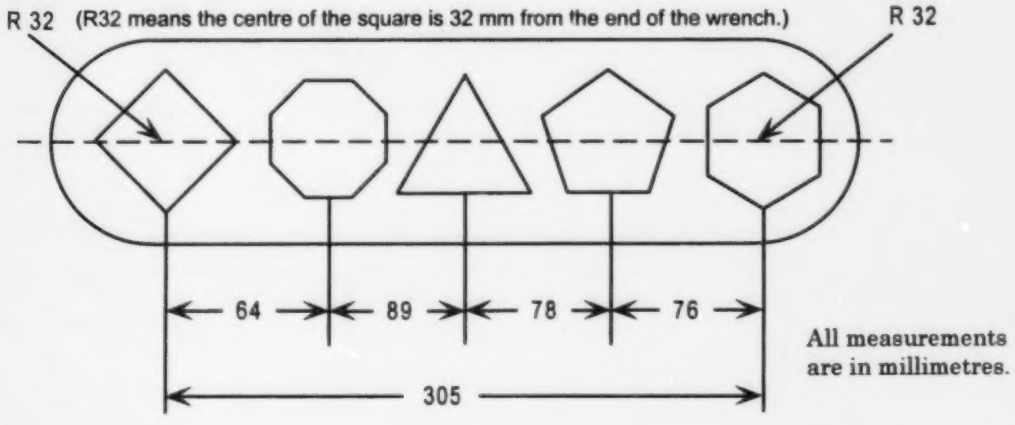
2. The opening between two parts is four-and-three-quarters thousandths (0.00475") of an inch. A shim is available, which is measured with a Vernier micrometer as 0.00335". Find the additional shim thickness to fill the opening.

3. A land developer is considering the purchase of a rectangular plot of land. The plot is shown on Municipal Land Titles as ½ mile long and 12 rods wide (1 rod = 16.5 ft.).
   a) Give the dimensions of the plot in feet.
   b) What is the plot's acreage?

      Use 1 acre = 4 x 40 rods or 1 rod x ½ mile.
      640 acres = 1 sq. mile
   c) A developer wants to subdivide the plot into 20 000 ft^2 lots. How many lots would be available to the developer?

Exercise 7 (continued)

4. The combination wrench shown below contains the following:
   - 25-mm square
   - octagon 35 mm across the flats (i.e., from one flat side to the opposite flat side)
   - isosceles triangle: 70 mm base and 50 mm side
   - an inscribed pentagon on a 35-mm diameter circle
   - hexagon 32 mm across the flats



   a) Calculate the area of material to make the wrench before the geometric shapes are cut.
   b) Calculate the area of material to make the wrench after the geometric shapes are cut.
   c) Calculate the mass of such a wrench if it were cut out of material having a density of 12.85 g/cm^3, and the thickness of material is 9 mm.

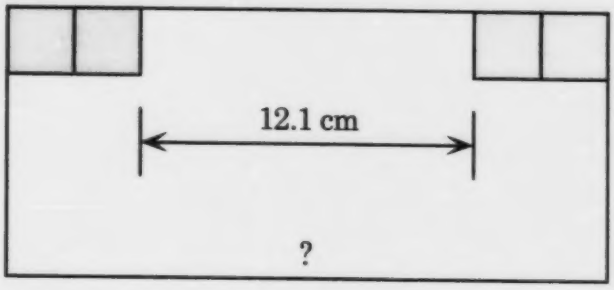
5. Determine the distance in inches that a machine tool travels for each of the different cuts (A, B, C, D, and E). Each distance is to be rounded to one decimal place.

   Give total distance for all cuts.

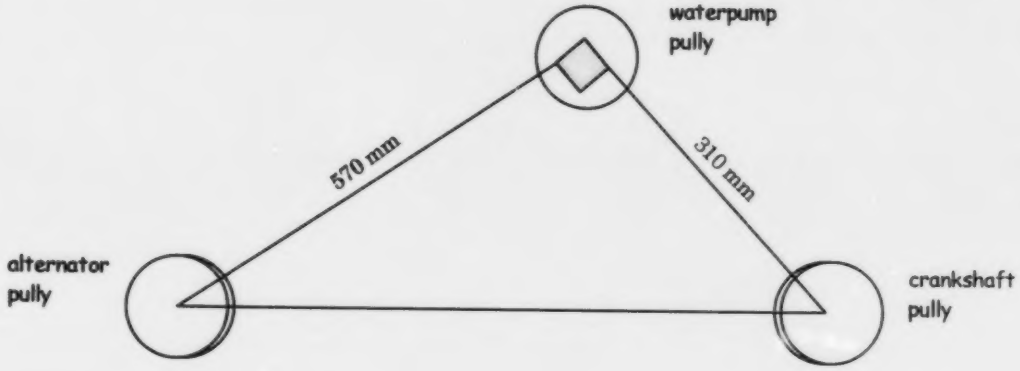
Cuts	RPM	Feed per Rev (inches)	Time (minutes)
A	900	0.005	1.5
B	424	0.008	2.25
C	368	0.015	6.75
D	336	0.062	5.75
E	128	0.062	25.25

Exercise 7 (continued)

6. The total contact area of a clutch disc with lining on both sides is 196 cm^2. The inside diameter of the lining is 12.1 cm. What is the outside diameter of the lining?



7. What is the centre-to-centre distance between the crankshaft pulley and the alternator pulley on the following drive layout?

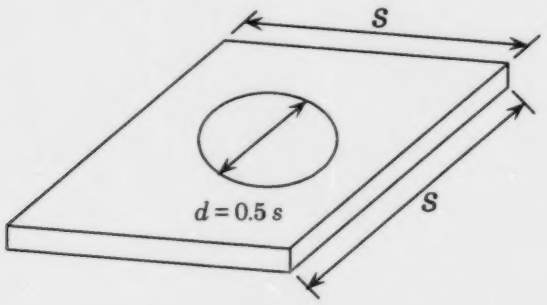


8. A gable roof is to be covered with asphalt shingles. Each side of the roof is 35 ft. by 21 ft. There are 27 shingles in a bundle that will cover a total of 33⅓ sq. ft.
   a) How many complete bundles of shingles must be ordered for the roof?
   b) How many shingles will be left over to shingle a dog house?

9. A 4-ft. by 8-ft. sheet of plywood is cut into strips each $1\frac{5}{16}$ in. wide by 4 ft. long. The saw cut is $\frac{1}{8}$ in. wide. How many strips can be cut from the sheet?

Exercise 7 (continued)

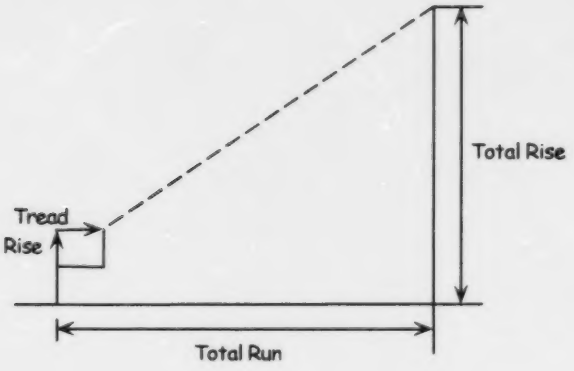
10. The area of a square panel is 705 600 mm^2. A round mirror, with a diameter, d, equal to one-half the length of one side s of the square panel, is set into the panel. What area of the panel requires tiling?



11. The mass of 144 similar hinges is 35.28 kg. What is the mass of one hinge?

12. The code requires that the total area of all openings for ventilation of an attic is to be $\frac{1}{300}$ of the area of the ceiling. In this particular attic, there are four openings with equal areas. The ceiling measures 9 600 mm by 11 500 mm. What is the area of each opening in square millimetres?

13. The stairs between two floors have a total rise of 266 cm. Each rise is 19 cm and each tread is 24 cm. What is the total run?



Exercise 8

Imperial measurement problems

1. A thickness wood planer advances wood at a rate of 12 ft./min. How long will it take to machine a piece of wood
   a) 18 in. long?
   b) 46 in. long?

2. A 4 ft. by 8 ft. sheet of plywood has a weight of 61.36 lb. What is the weight in ounces (16 ounces/lb) of a piece of the same plywood measuring
   a) 5 in. by 9 in.?
   b) 3 ft. by 7 ft.?

3. The footing for a house measured a total of 130 yds. long, 2'–6" wide, and 1'–6" thick. The concrete mix required for one cubic yard was 5⅓ bags of cement, 9 ft.^{3} of sand, and 15 ft.^{3} of aggregate.
   a) How many whole bags of cement were used for the footing?
   b) One bag of cement has a mass of 80 lb. How many tons of cement were used for the footing (1 ton = 2000 pounds)?
   c) How many cubic yards of sand were used for the footing?
   d) How many cubic yards of gravel were used for the footing?

4. A table top requires 11 wooden dowels that are $2\frac{15}{16}$ long. The dowelling is purchased in 4 ft. lengths.
   a) How many dowels can be cut from one length of dowelling?
   b) How many table tops can be finished with 19 lengths of dowelling?

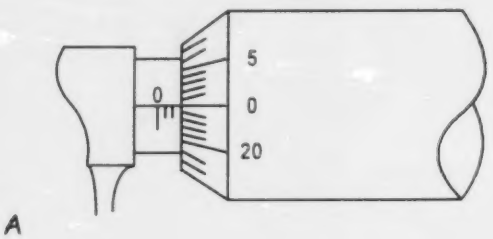
5. A rectangular printed circuit board is 1.80 in. by 3.45 in.
   a) How many circuit boards can be cut from a sheet 3 ft. by 4 ft.? (Note: Several answers are possible.)
   b) A sheet is 2 ft. 4 in. wide. What is the minimum length, to the nearest inch, required to produce 32 boards of the same 1.8 in. by 3.45 in. dimensions?

6. An auto installation company uses a 250 ft. spool of shielded cable to connect the car stereo to its antenna. A length of 56 in. is to be used. Each length is cut from a spool of 250 ft.
   a) How many lengths can be cut from the 250 ft. spool?
   b) If only 37 lengths were cut for a particular project, what length would remain on the spool in feet and inches?

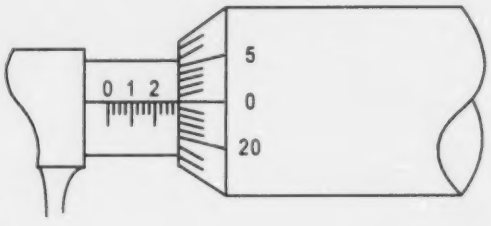
Exercise 9

Precision measurement problems

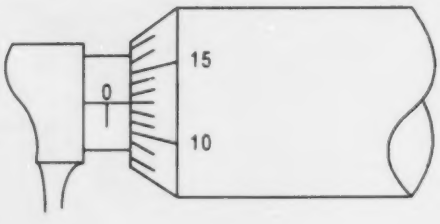
1. Determine the linear dimensions indicated at A, B, C, D, E, and F on a standard inch micrometer.



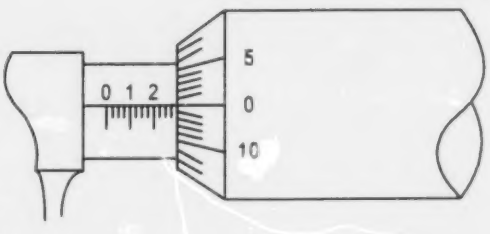
A



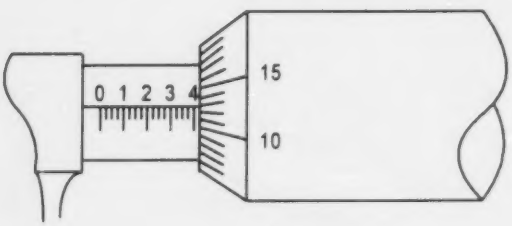
B



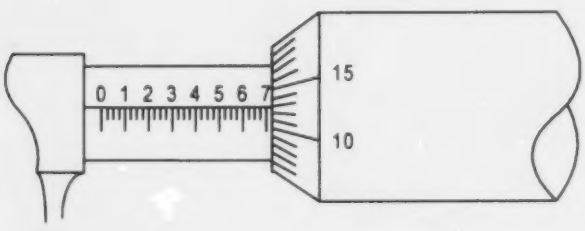
C



D



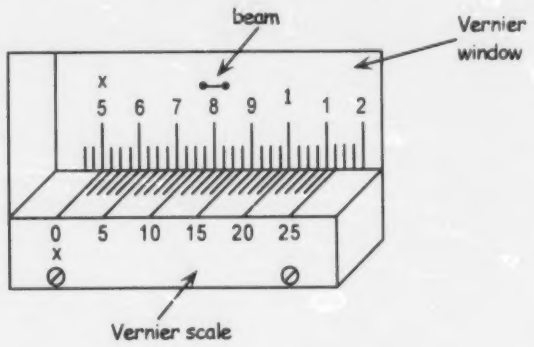
E



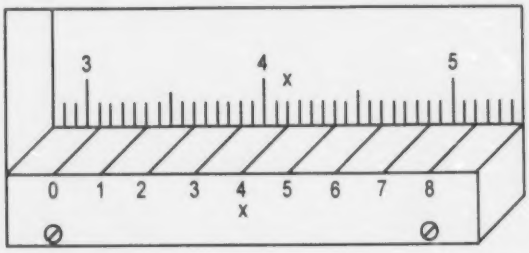
F

Exercise 9 (continued)

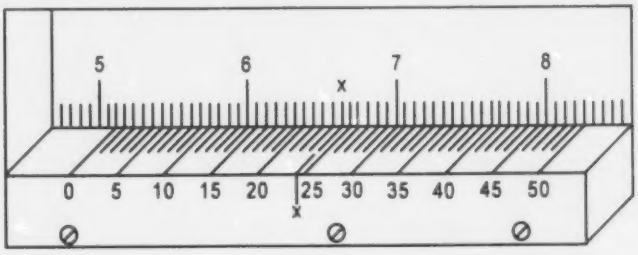
2. Determine the reading on this 25 division Vernier caliper.



3. Determine the reading on this 8 division Vernier caliper.

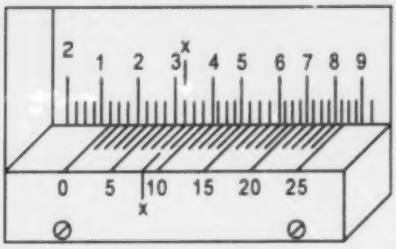


4. Determine the reading on this 50 division Vernier caliper.

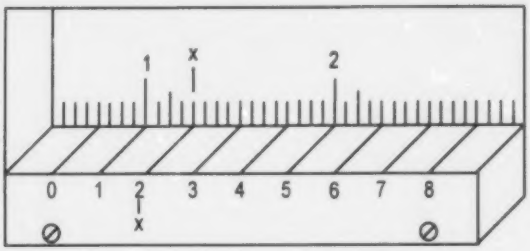


Exercise 9 (continued)

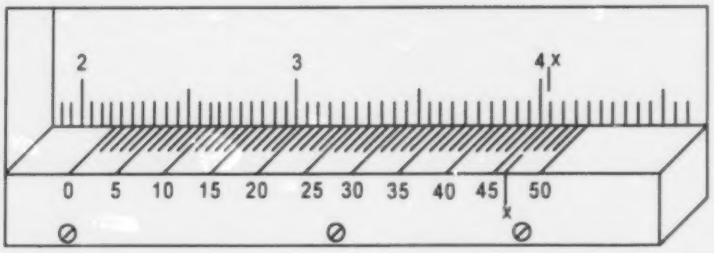
5. Determine the reading on this 25 division Vernier caliper.



6. Determine the reading on this 8 division Vernier caliper.

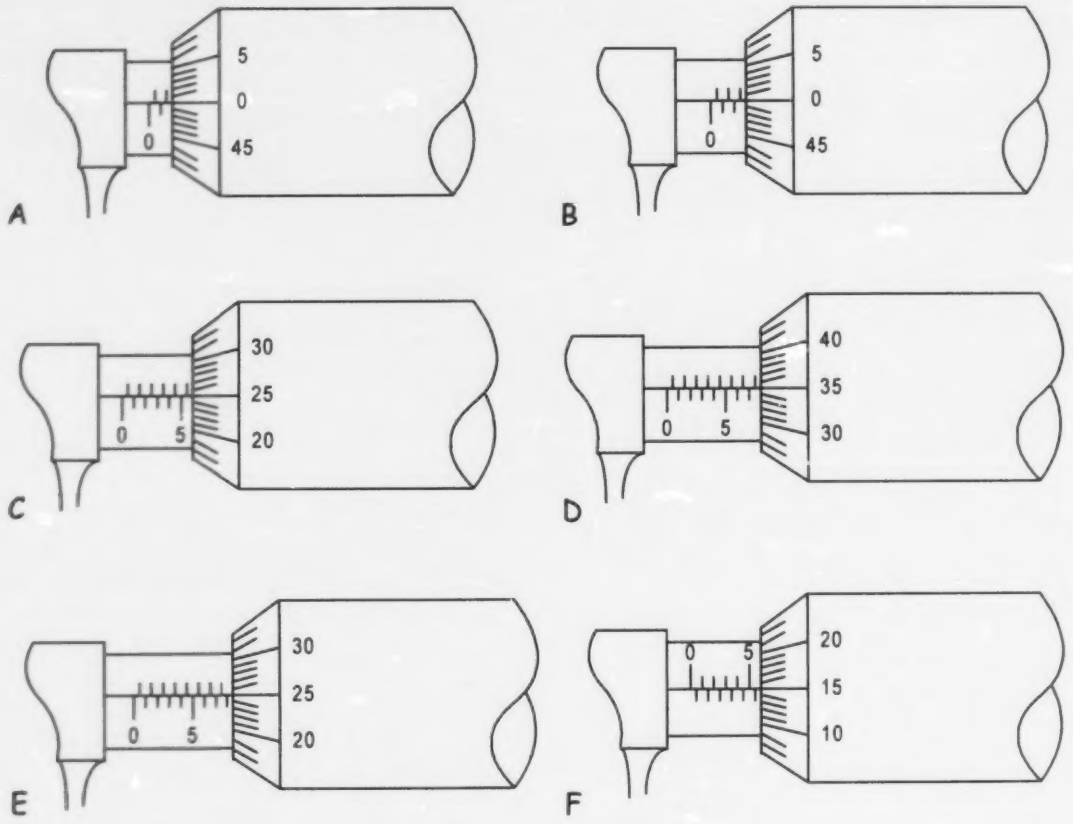


7. Determine the reading on this 50 division Vernier caliper.



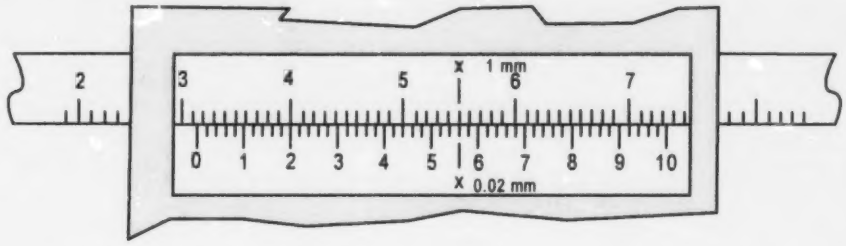
Exercise 9 (continued)

8. Determine the linear dimensions on the following standard metric micrometer settings.

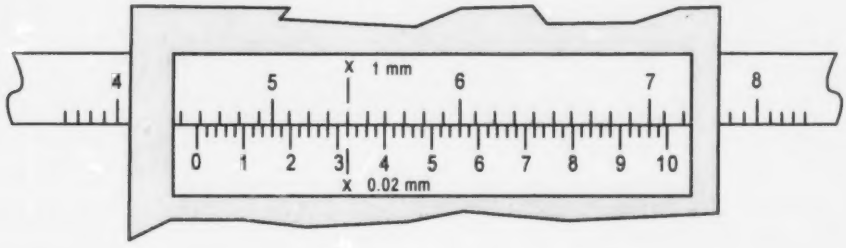


Exercise 9 (continued)

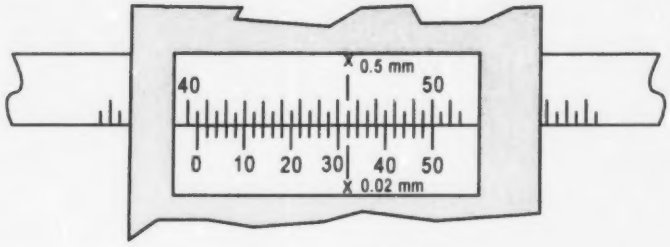
9. Determine the reading on this metric Vernier caliper.



10. Determine the reading on this metric Vernier caliper.

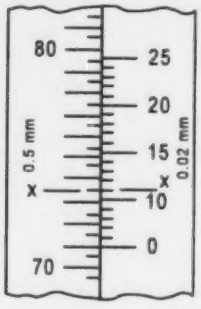


11. Determine the reading on this metric Vernier caliper.

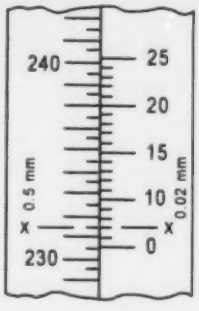


Exercise 9 (continued)

12. Determine the Vernier height gauge settings.

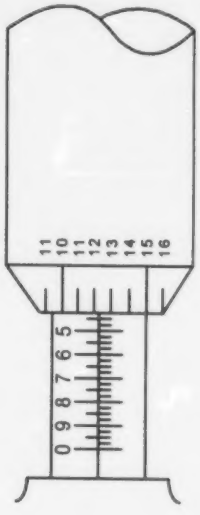


A



B

13. Determine the depth reading of the 0.001" depth micrometer with a 2 inch extension.



14. How long will it take to drill a piece of work 25.4 mm thick, if the drill is turning at 220 r/min and the feed is 0.08 mm/r?

$$\text{time} = \frac{\text{depth of hole}}{\text{feed} \times \text{r / min}}$$

Unit H
Measurement Technology
Solutions

Solutions are provided only for Exercise 2

Exercise 2 Solutions

1. a) 600 cm
   b) 6500 mm
   c) 19 000 mL
   d) 3000 mm
   e) 64 000 m
   f) 350 m
   g) 0.853 L
   h) 12 400 g
   i) 3.423 m
   j) 2.25 kg
   k) 1.75 m

2. a) $10\ 000\ cm^2$
   b) $1\ 000\ 000\ cm^3$
   c) $1\ 000\ 000\ mm^2$
   d) $1\ 000\ 000\ 000\ mm^3$
   e) $35\ 000\ cm^2$
   f) $2\ 800\ 000\ cm^3$
   g) $1.256\ m^2$
   h) $1.85\ cm^2$
   i) $0.000\ 135\ m^3$

3. a) 345 cm or 3.45 m
   b) 4025 g or 4.025 kg
   c) 160 mm or 16 cm
   d) 2625 mL or 2.625 L
   e) 22 625 m or 22.625 km
   f) 2422 mm or 2.422 m

Exercise 2 Solutions (continued)

1. a) 63 in.

   b) 87 in.

   c) 18 480 ft.

   d) 6.25 ft.

   e) $\frac{1}{3}$ ft.

   f) 2 yd.

2. a) 11 ft. 5 in.

   b) 1 yd. 2 ft. 8 in.

   c) 10 ft. 4 in.

   d) 5 yd. 2 ft.

3. a) $2\frac{5}{16}$ in.

   b) $1\frac{82}{128}$ in.

   c) $\frac{17}{16}$ in. or $1\frac{1}{16}$ in.

   d) $4\frac{1}{16}$ in.

   e) $\frac{3}{128}$ in.

   f) $\frac{35}{128}$ in.

   g) $\frac{25}{128}$ in.

   h) $2\frac{99}{128}$ in.

   i) $3\frac{53}{128}$ in.

   j) $1\frac{65}{128}$ in.

Unit I
Trigonometry

These exercises contain questions of various complexity. Some are simple projects. Not every student will likely complete all questions. Teachers are encouraged to look over the problems, assign ones they like or assign students different problems. For assessment, students can hand in their solutions or present them to the class.

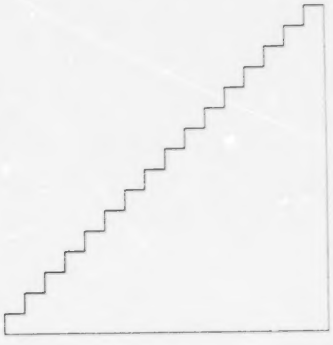
Exercise 1

Problems with two triangles including angles of elevation and depression

1. Use your angular measuring device to measure the angle of elevation to the basketball hoop in the gym. Stand at the free throw line. Measure the distance to the front of the hoop using a tape measure. Verify that the hoop is 10 feet tall.

2. Have another student measure the length of your stride. Now go outside and pace off 20 strides from the outside wall of your school. Measure the angle of elevation to the top of the school and calculate the height of the school.

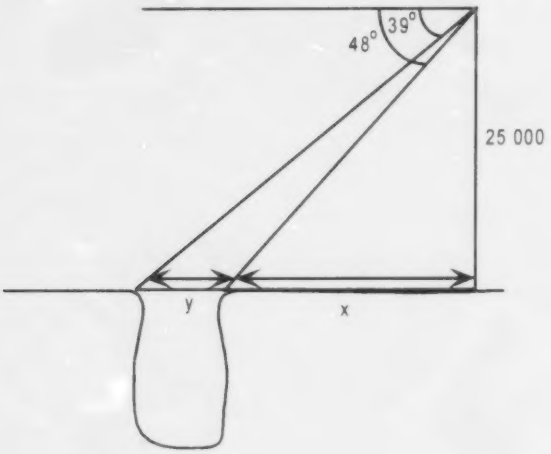
3. The staircase below is drawn such that 1 cm = 0.75 m on the actual stairs.
   a) What are the actual horizontal and vertical dimensions of the staircase?
   b) What is the angle of the staircase?
   c) What is the length of the staircase?



4. a) Draw a diagram of a 2-litre milk carton. Label the angle θ as the angle of elevation of the top part.
   b) Write directions to find θ using only a ruler and a calculator or table. Write directions to verify the value with a protractor. Use diagrams if necessary.
   c) Get a 2-litre milk carton. Follow your directions in (b) to find θ. Verify angle θ using your directions with a protractor.

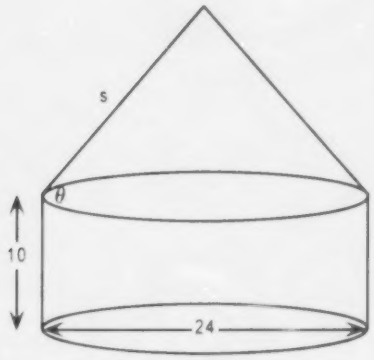
Exercise 1 (continued)

5. A plane is flying at 25 000 feet. A surveyor spots two sides of a canyon. The angles of depression to each side are 39° and 48°. Use the diagram to calculate the distance across the canyon.



6. A helicopter is flying at 175 m and spots a speeding car at an angle of depression of 50°. A police car chasing the speeding car is at an angle of depression of 60° from the helicopter.
   a) Draw a diagram of the scenario to scale so that 1 cm = 20 m.
   b) Use the diagram to measure the distance between the police car and the speeding car.
   c) Use trigonometry to verify your answer in part (b).

7. a) A farmer phones and asks for your assistance. He is building a cylindrical grainery with a cone on top as shown below. He intends to build the entire grainery in his workshop that has a doorway 15 feet high. He also tells you that the sled he will move it on is $8\frac{3}{8}$ inches high. The base of the grainery cannot be greater than 24 feet in diameter and the sheets of metal he is using are 10 feet wide, thus limiting the height of the cylindrical part. His question is: What angle of elevation (θ) should he make the cone on top to give the greatest volume and yet still fit under the door?



Exercise 1 (continued)

b) Calculate the volume of the grainery if the farmer decides to round his total height down to the nearest foot to ensure that the grainery will fit. Answer in cubic yards.

c) If each sheet of metal is 10 feet wide by 12 feet long and costs $139.50, what will the total metal cost be? You will have some waste metal left over. You should use the total area in your figures and order one extra sheet for mistakes.

c) If concrete costs $140 per cubic yard and the base of the grainery is made of concrete and must be 4 inches thick, what will be the total cost of the grainery?

8. A cottage owner is installing a TV antenna on her roof. The TV antenna is 4 m high and the support wires will be attached 0.5 m below the top of the antenna. The cottage is a square that is 6 m on each side. The peak of the roof is in the middle of the cottage as shown.

a) Draw the top view of the cottage. Show the measures of the sides of the cottage and the lines joining the corners to the point directly below the TV antenna.

b) How would the cottage owner measure the height of the roof? How would she measure the angle of the roof along the corner ridges? Write a set of directions to do both.

c) The cottage owner has followed your directions and discovered that the roof is 2.2 m high and the angle of the roof is 27°. Support wires will be connected from the TV tower to each corner of the roof. Calculate the length of each wire. Show a diagram showing a side view of the roof ridge, the support wire, and the TV antenna.

d) Support wire costs $4.25/m. Assume that you will waste 30 cm each time you connect the wire to the roof or tower, and that wire comes in lengths of 1 m. What would the total cost be including GST and PST?

9. Have a person measure your stride length and the height of your eye. Now go outside and pick any tall building. Measure the angle of elevation to the building from any point you wish to stand. Now take 30 strides back and measure the angle of elevation. Calculate the height of the building. Be sure to account for stride length and your own height.

Exercise 2

Extending concept of sine and cosine for $\theta > 90°$

1. a) Using a calculator, find

   i) $\sin 30° =$ _____ $\sin 150° =$ _____

   ii) $\sin 45° =$ _____ $\sin 135° =$ _____

   iii) $\sin 10° =$ _____ $\sin 170° =$ _____

   b) What is the relationship between each pair of angles (i.e., how are $30°$ and $150°$ related)?

   c) Come up with a rule for the sine of angles between $90°$ and $180°$.

2. a) Using a calculator, find

   i) $\cos 30° =$ _____ $\cos 150° =$ _____

   ii) $\cos 45° =$ _____ $\cos 135° =$ _____

   iii) $\cos 10° =$ _____ $\cos 170° =$ _____

   b) What is the relationship between each pair of angles (i.e., how are $30°$ and $150°$ related)?

   c) Come up with a rule for the cosine of angles between $90°$ and $180°$.

3. *Spreadsheet activity:* Set up a spreadsheet to look like the one below with all angles from $0°$ to $180°$ (multiples of 5) filled in. Use *fill down* and be sure to use the formula =SIN((3.14/180°)*A2) to convert to radians. When you are done, make a line chart or scatterplot of the entire chart with the angle as the X-series and the sin and cos values as the two Y-series. Sketch what it looks like. Now *fill down* to include angles up to $360°$ again in multiples of 5. What does the chart look like now? Add the tan column beside the cos column. Make a new chart. You may have to leave out the tan values for $80°$, $90°$, $100°$ and $170°$, $180°$, $190°$. Explain why this is necessary. What other patterns do you see in the spreadsheet?

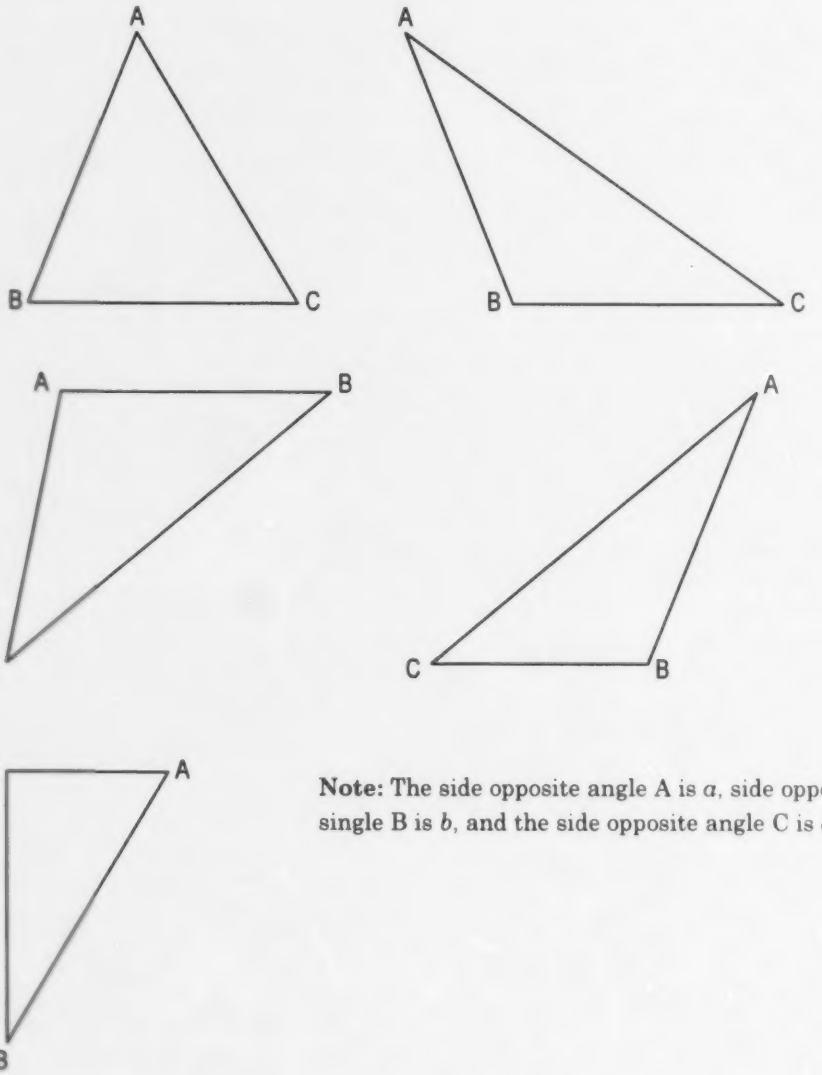
	A	B	C
1	angle	sin	cos
2	0	0	1
3	5	0.09	1.00
4	10	0.17	0.98
5			
6			
7	170	0.18	-0.98
8	175	0.09	-1.00
9	180	0.00	-1.00

Exercise 3

Sine Law Activity

Purpose: To discover the sine law

Directions: Carefully measure all the angles (to the nearest degree) and all of the sides (to the nearest millimetre) in all the triangles. Record your measurements. Then create a spreadsheet to look like the one given on the next page. Write what you think is the **sine law** based on your observations.



Note: The side opposite angle A is a, side opposite single B is b, and the side opposite angle C is c.

Spreadsheet

	A	B	C	D	E	F	G	H	I	J	K	L
1	A	sin A	a	a/sin A	B	sin B	b	b/sin B	C	sin C	c	c/sin C
2												
3												
4												
5												

Note: To get the sine of A the formula is: =sin((3.14/180°)*A)

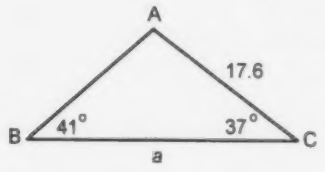
What relationship do you see among columns a/sinA, b/sin B, and c/sin C? You may need to round them to 1 decimal place to see the relationship.

Write the sine law.

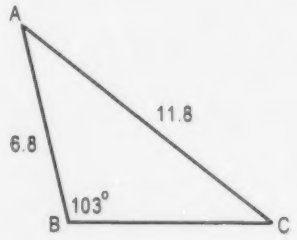
Exercise 4

Sine law problems

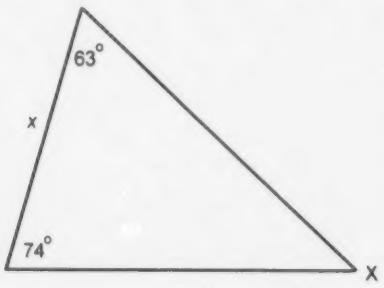
1. Solve for *a*.



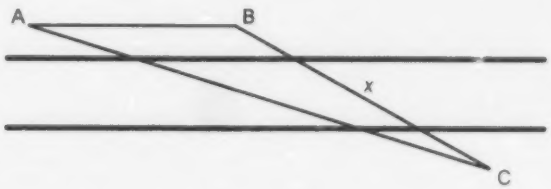
2. Solve for C.



3. Solve for *x*.

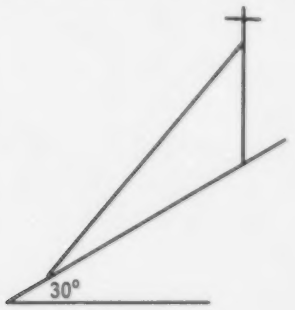


4. A surveyor stands at point A and sights point C at 17° S of E. He then takes 20 paces along the river to point B. Here point C is sighted at 29° S of E. If each pace is 0.95 m,
   a) find the distance from B to C
   b) how many more paces would the surveyor take before being directly across from C?

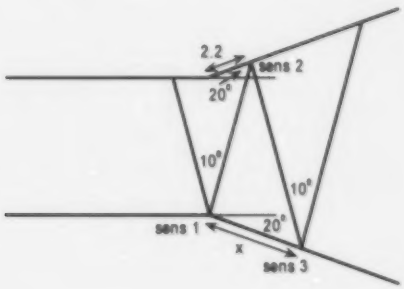


Exercise 4 (continued)

5. A TV antenna is being mounted as shown. The antenna is 2.5 m tall and the support wire is to be mounted 0.2 m from the top. The roof slopes up at 30° and the angle the support must make with the horizontal is 50°. If the support wire is sold in multiples of 50 cm for $0.79 per metre, calculate the total cost of wire including GST (7%) and PST (7%).

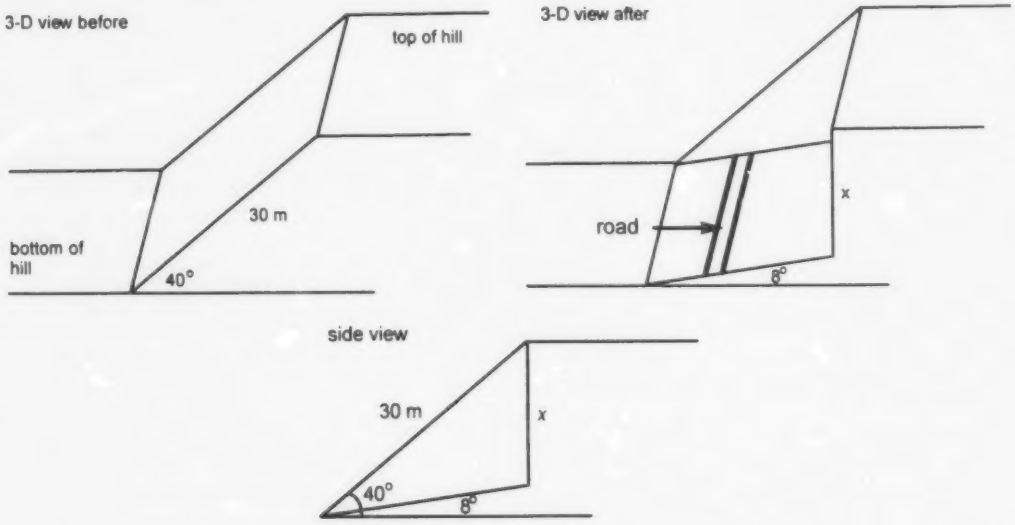


6. A security system is being designed for a widening hallway as shown. Each sensor has an angle of action of 10° and a range of sensing of 150 m. Determine the distance from the corner the third sensor is placed so that there is no overlap. The hall widens at a 20° angle.

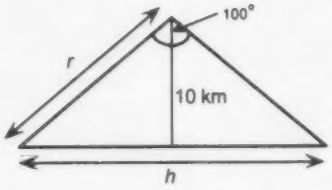


Exercise 4 (continued)

7. A hill has a 40° incline and is 30 m from the base of the top. The hill is to be cut down to a road with an 8° incline. How many vertical metres (x) will be removed? If the hill is 25 m long and it costs \$17.50/m^3 to remove dirt, calculate the cost of removing the dirt including PST (7%) and GST (7%).



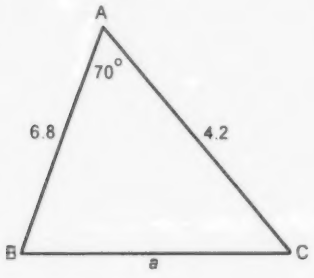
8. A communications satellite is placed 10 km above Earth. The angle of its signal is 100° as shown.
   a) What is its transmitting range (r) in kilometres?
   b) What horizontal distance (h) on land will a satellite cover?
   c) How far away horizontally should a second satellite be placed if there is to be a 1.5 km distance of overlap?

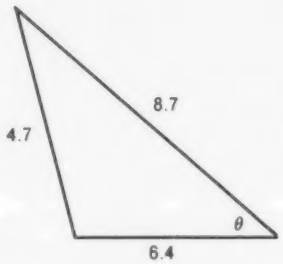


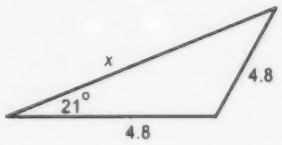
Exercise 5

Cosine law problems

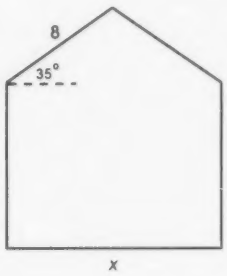
1. Solve for the missing side or angle.



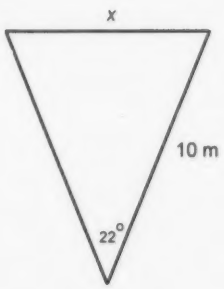




2. A carpenter building a shed wishes to make the roof at an angle of 35°. If the rafters are pre-fabricated to be 8 feet in length, how wide can the shed be in feet and inches (nearest quarter inch)?

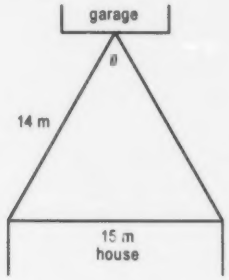


3. A motion detector has an angle of 22°. If the sensor has a range of 10 m, what is the width of the sensing area at its widest section?



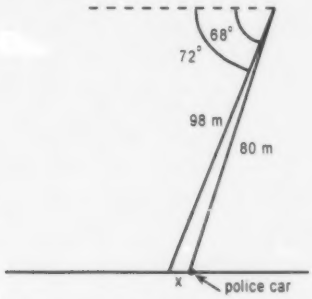
Exercise 5 (continued)

4. A motion sensor has the capability of adjusting its angle of action, but the range of sensing remains 14 m. The sensor is mounted on a farm garage and the farmhouse is 15 m wide. At what angle (θ) should it be set so that the entire side of the house is in the action zone, if the corner of the house is 14 m away from the sensor?

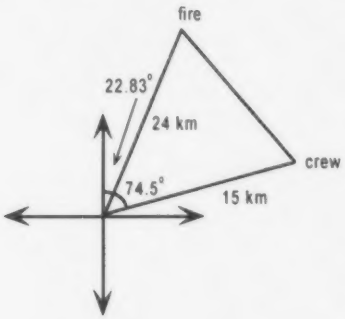


5. A police helicopter spots a speeding car at an angle of depression of 68°. The police car chasing it is at an angle of depression of 72°. If the helicopter can use sonar to determine the car is 98 m away and the police cruiser is 80 m away, how far apart are they?

   Advanced: If the speeding car is travelling at 38.9 m/s and the cruiser at 41.7 m/s, how many seconds will it be before the cruiser catches it?

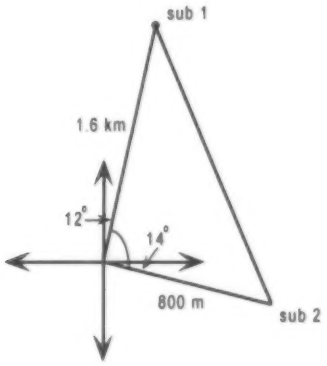


6. A forest fire spotter is in a tower. She spots a fire at a heading of 22.83° E of N and the fire crew on a road at the heading 74.5° E of N. The crew is 15 km from the tower and the fire is estimated to be 24 km from the tower. How far is the crew from the fire? It they can travel at 3 km/h, how long will it take to get to the fire, assuming the fire does not spread toward them?

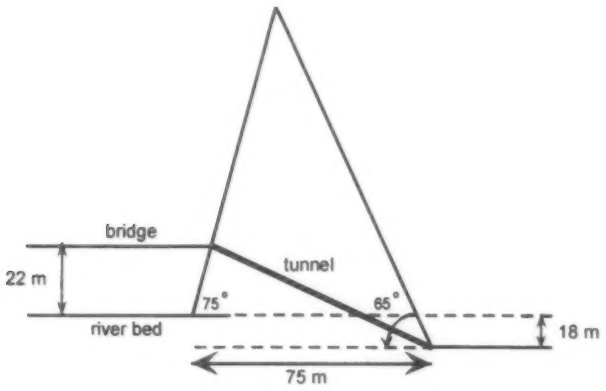


Exercise 5 (continued)

7. A nuclear submarine finds another submarine at 12° E of N. It uses sonar to determine that the submarine is 1.6 km away. A second sub is spotted at 14° S of E and is 800 m away. How far apart are the other two subs?



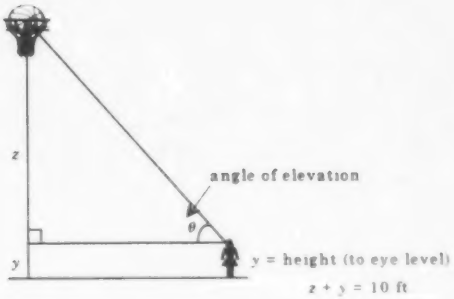
8. A railway tunnel is to be built through a mountain as shown. There is a river on the west side of the mountain whose bed is 18 m above the ground on the east side of the mountain. The west side of the mountain has a rise of 75° while the east side is 65°. The bridge above the river is 22 m above the river bed and the base of the mountain is 75 m wide. Determine the length of the tunnel, x. If the tunnel has a diameter of 4 m, determine the volume of rock that will be removed.



Unit 1
Trigonometry
Solutions

Exercise 1 Solutions

1. Possible student diagram:



2. Students will need to account for their own height in this question.

3. a) $h = 3.9$ m, v = 2.175 m
   b) $\theta = 29.1°$
   c) $x = 4.47$ m

4. a)

   b) Students should be measuring the distances x and y, then using tan to solve for θ. To verify a student can simply measure with a clear protractor with the flat part of the protractor horizontal.
   c) Answers may vary depending on brand.

5. Solution:

$$90° - 48° = 42°$$

$$\tan 42° = \frac{x}{25\,000}$$

$$x = 25\,000 \tan 42° = 22\,510.101 \text{ feet}$$

$$90° - 39° = 51°$$

$$\tan 51° = \frac{22\,510.101}{25\,000}$$

$$y = 25\,000 \tan 51° = 22\,510.101 = 8362.3 \text{ feet}$$

Exercise 1 Solutions (continued)

6. c) $90^\circ - 60^\circ = 30^\circ$

$$\tan 30^\circ = \frac{x}{175}$$

$$x = 175 \tan 51^\circ = 101.0363 \text{ m}$$

$90^\circ - 50^\circ = 40^\circ$

$$\tan 40^\circ = \frac{101.0363 + y}{175}$$

$$y = 175 \tan 40^\circ - 101.0363 = 45.8 \text{ m}$$

7. The maximum height of the cone portion will be

$15' - 10' = 60''$ $\qquad$ $60'' - 8\frac{3}{8}'' = 51.625''$

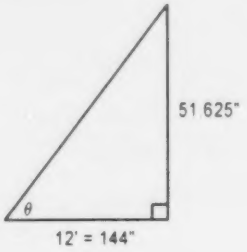
a) This maximum height will yield the maximum volume. To solve for θ:

$$\tan \theta = \frac{51.625}{144}$$

$$\tan \theta = 0.035\,850\,69$$

$$\theta = \tan^{-1}(0.358\,506\,9)$$

$$\theta = 19.7^\circ$$



b) To actually find the volume, the farmer would round down to a cone height of 4 feet. The volume produced is then

$$V_{\text{cylinder}} = V\pi r^2$$

$$= \pi \times 12^2 \times 10$$

$$= 4521.6 \text{ ft}^3$$

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times 12^2 \times 4$$

$$= 602.88 \text{ ft.}^3$$

$$V_{\text{total}} = 4521.6 + 602.88 = 5124.48 \text{ ft.}^3$$

$$5124.48 \text{ ft.}^3 \div 27 = 189.8 \text{ yd.}^3$$

Exercise 1 Solutions (continued)

c) It is suggested that students calculate the total area then divide by the area of each sheet, round up and then buy one extra.

Area each sheet = $10 \times 12 = 120$ ft.^{2}

Area bottom = circumference × height

$= 2\pi h$

$= 2 \times \pi \times 12 \times 10$

$= 753.6$ ft.^{2}

Sheets needed =

$753.6 \div 120 = 6.28$ sheets

Area cone = πrs

$s^2 = 4^2 + 12^2$

$s = 12.649111$

$A = \pi \times 12 \times 12.649111$

$A = 476.61$

Sheets needed =

$476.61 \div 120 = 3.97$ sheets

Total sheets = 6.28 + 3.97 = 10.25 sheets
Round up and add 1 = 12 sheets x \$139.50 = \$1674.00

d) Volume of base = $\pi r^2 h$

$= \pi \times 12^2 \times \frac{4}{12}$

$= 150.72$ ft.^{2}

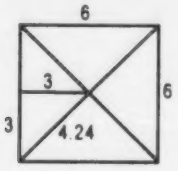
150.72 ft^3 ÷ 27 = 5.58 yd.^{3}

Cost: 5.58 × \$140 = \$781.50

Total cost of grainery = \$781.50 + \$1674 = \$2455.50.

8. Students may have a difficult time solving this problem because they cannot visualize it. It is suggested that students build a model out of cardboard.

a) The top view looks like this:



$3^2 + 3^2 = 18$

$\sqrt{18} = 4.24$

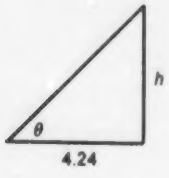
Note: This is a view showing the top of the house without the roof on. The diagonal lines are not the roof edges.

b) The cottage owner can measure the height of the roof by doing the following:

i) Measure the height of the roof inside the house and add a little for the width of the roof.

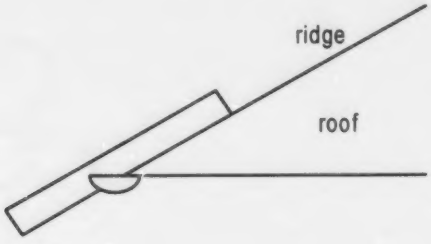
ii) Use an angular measuring device and trig

iii) Use Pythagoras to find the distance to the point in middle (above). Then, find the angle of the roof along a ridge and solve for h.

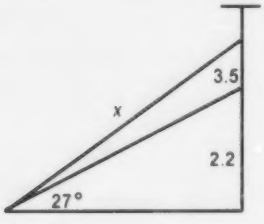


Exercise 1 Solutions (continued)

c) The cottage owner can find the angle of the roof ridge by placing a board on the ridge overhanging. Then, she can place a protractor upside down flush with the roof and read the angle.



d) The diagram students should use looks like this:



Solution:

$x^2 = (2.2 + 3.5)2 + 4.242$
$x^2 = 50.4676$
$x = 7.104$ m

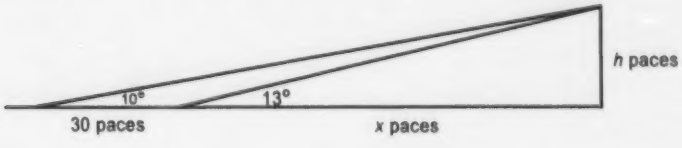
Each wire is 7.104 m long plus waste = 7.104 + 0.3 + 0.3 = 7.7 m long
Four wires = 7.7 x 4 = 30.6 m.

The cottage owners must buy 31 m of wire x $4.25	= $131.75
plus tax	= $ 18.45
Total	= $150.20

9. Suppose the following measurements:

stride length = 8 m
eye height = 1.75 m
first angle of elevation = 13°
second angle of elevation = 10°

The following is the solution:



$\tan 13^\circ = \frac{h}{x}$ $\qquad$ $\tan 10^\circ = \frac{h}{x+30}$

$h = x \tan 13^\circ$ $\qquad$ $(x+30) \tan 10^\circ = h$

Exercise 1 Solutions (continued)

Since h's are equal:

$x \tan 13 = (x + 30) \tan 10°$

To avoid factoring and manipulation problems, use the actual trigonometric values:

$0.231x = (x + 30).176$
$0.231x = 0.176x + 5.28$
$0.055x = 5.28$
$x = 96$ paces

Now substitute x into the equation:

$$\tan 13° = \frac{h}{96}$$

$h = 96 \tan 13°$
$h = 22.16$ paces

Now convert to metres:

22.16×0.8 m = 17.73 m

Add the height of observer:

$17.73 + 1.75 = 19.48$ m

Exercise 2 Solutions

1. a) i) 0.5 0.5
   ii) 0.707 0.707
   iii) 0.174 0.174

   b) Each pair of angles is supplementary. The angles add up to 180°.

   c) The sine of an angle > 90° is equal to the sine of its supplement or $\sin \theta = \sin (180° - \theta)$.

2. a) i) 0.866 –0.866
   ii) 0.707 –0.707
   iii) 0.984 –0.984

   b) Each pair of angles is supplementary. The angles add up to 180°.

   c) The cosine of an angle > 90° is equal to the negative cosine of its supplementary angle or $\cos \theta = -\cos (180° - \theta)$.

3. Answers will vary.

Exercise 3 Solutions

Students will have a difficult time spotting the relationship among columns a/sin A, b/sin B, and c/sin C. It is suggested that they round to one decimal place or nearest whole number. Students can hide any column by changing its column width to zero. For example, to compare columns D, H, and L, change all other column widths to zero.

Exercise 4 Solutions

1. $\dfrac{\sin 41^\circ}{17.6} = \dfrac{\sin 37^\circ}{x}$

$x = \dfrac{17.6 \sin 37^\circ}{\sin 41^\circ}$

$x = 16.1$

2. $\dfrac{\sin 103^\circ}{11.8} = \dfrac{\sin C}{6.8}$

$\sin C = \dfrac{6.8 \sin 103^\circ}{11.8}$

$C = \sin^{-1}(0.5615013)$

$C = 34.2^\circ$

3. $x = 180 - (74 + 63)$

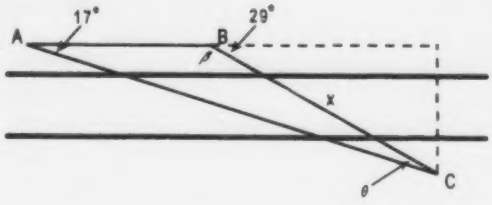
$x = 43^\circ$

$\dfrac{\sin 43^\circ}{x} = \dfrac{\sin 74^\circ}{10.2}$

$x = \dfrac{10.2 \sin 43^\circ}{\sin 74^\circ}$

$x = 7.2$

4. a)



$\beta - 180^\circ - 29^\circ$

$\beta = 151^\circ$

$\theta = 180 - (151 + 17)$

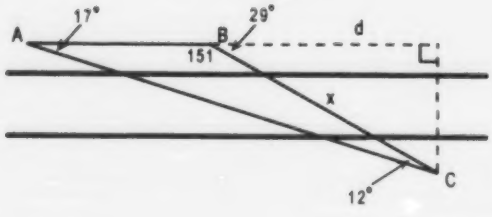
$\theta = 12^\circ$

$\dfrac{\sin 17^\circ}{x} = \dfrac{\sin 12^\circ}{20}$

$x = 28.1$ paces

28.1 paces × 0.95 m = 26.7 m

b)



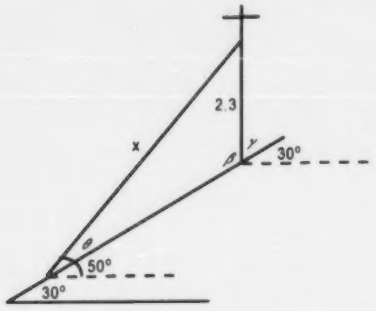
$\cos 29^\circ = \dfrac{d}{28.1 \text{ paces}}$

$d = 28.1 \times \cos 29^\circ$

$d = 24.6$ paces

Exercise 4 Solutions (continued)

5.



$\theta = 50° - 30°$

$\theta = 20°$

$\gamma = 90 - 30°$

$\gamma = 60°$

$\beta = 180° - 60°$

$\beta = 120°$

$$\frac{\sin 120°}{x} = \frac{\sin 20°}{2.3}$$

$$x = \frac{2.3 \sin 120°}{\sin 20°}$$

$x = 5.82$ m

buy 6 m of wire

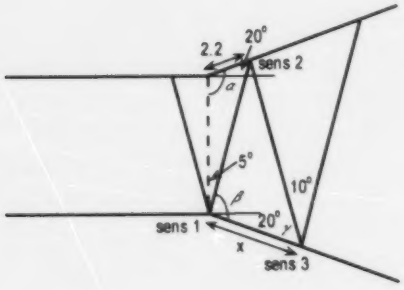
need 12 lengths at \$0.79 = \$9.48

PST = \$0.66

GST = \$0.66

Total = \$10.80

6.



$\alpha = 90° + 20°$

$= 110°$

$\beta = 90° - 5 + 20°$

$= 105°$

$$\frac{\sin 110°}{y} = \frac{\sin 5°}{2.2}$$

$$y = \frac{2.2 \sin 110°}{\sin 5°}$$

$y = 23.71988$ m

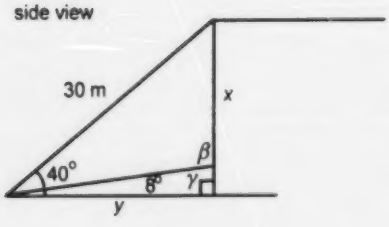
$\gamma = 180° - 10° - 105°$

$\gamma = 65°$

$$\frac{\sin 10°}{x} = \frac{\sin 65°}{23.71988}$$

$x = 4.54$ m

7.



$a = 90° - 8°$

$= 82°$

$b = 180 - a$

$= 98°$

$$\frac{\sin 98°}{30} = \frac{\sin 32°}{x}$$

$$x = \frac{30 \sin 32°}{\sin 98°}$$

$x = 16.05$ m

$$\cos 40° = \frac{y}{30}$$

$y = 30 \cos 40°$

$= 22.98$ m

area $= 0.5bh$

$= 0.5xy$

$= 0.5(16.05)(22.98)$

$= 184.4$ m^2

V = area × 25 m

$= 184.4 \times 25$

$= 4610$ m^3

cost $= 4610 \times \$17.50$

$= \$80\ 675$

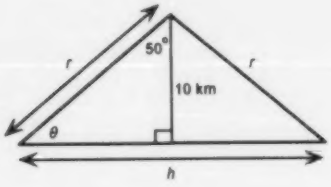
GST = \$ 5647.25

PST = \$ 5647.25

Total = \$91 969.50

Exercise 4 Solutions (continued)

8.



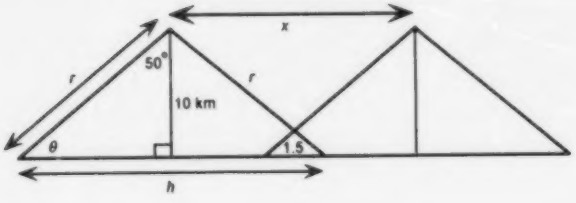
a) $\cos 50^\circ = \dfrac{10}{r}$ $\qquad \theta = \dfrac{180^\circ - 100^\circ}{2}$

$r = \dfrac{10}{\cos 50^\circ}$

$r = 15.557238 \text{ km}$

b) $\dfrac{\sin 100^\circ}{h} = \dfrac{\sin 40^\circ}{15.557238}$

$h = 23.8 \text{ km}$



c) $x = 2 \times \dfrac{h}{2} - 1.5$

$x = 22.3 \text{ km}$

Exercise 5 Solutions

1. a) $a^2 = 6.8^2 + 4.2^2 - 2(6.8)(4.2)\cos 70^\circ$

$a^2 = 44.343809$

$a = 6.7$

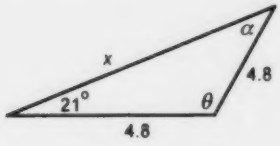
b) $4.7^2 = 8.7^2 + 6.4^2 - 2(8.7)(6.4)\cos\theta$

$\cos\theta = \dfrac{4.7^2 - 8.7^2 - 6.4^2}{-2(8.7)(6.4)}$

$\theta = \cos^{-1}(0.8491379)$

$\theta = 31.9^\circ$

c)



Since this is an isosceles triangle,

$\alpha = 21^\circ$

and

$\theta = 180^\circ - 21^\circ - 21^\circ$

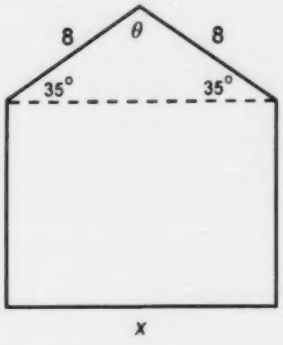
$= 138$

$x^2 = 4.8^2 + 4.8^2 - 2(4.8)(4.8)\cos 138^\circ$

$x^2 = 60.348381$

$x = 7.8$

2.



$\theta = 180 - 35 - 35$

$\theta = 110$

$x^2 = 8^2 + 8^2 - 2(8)(8)\cos 110^\circ$

$x^2 = 215.55716$

$x = 14.681865$ feet

$\dfrac{1 \text{ foot}}{12 \text{ in.}} = \dfrac{0.681865 \text{ feet}}{x \text{ in.}}$

$x = 8.1823787$ in.

Round this to nearest quarter inch = 8.25 inches

x = 14' 8 1/4"

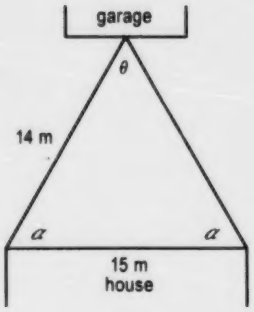
3. $x^2 = 10^2 + 10^2 - 2(10)(10)\cos 22^\circ$

$x^2 = 14.563229$

$x = 3.82$ m

Exercise 5 Solutions (continued)

4.



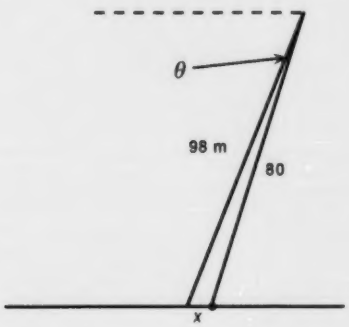
$$15^2 = 14^2 + 14^2 - 2(14)(14)\cos\theta$$

$$\cos\theta = \frac{15^2 - 14^2 - 14^2}{-2(14)(14)}$$

$$\theta = \cos^{-1}(0.426024)$$

$$\theta = 65^\circ$$

5.



$$\theta = (90^\circ - 68^\circ) - (90^\circ - 72^\circ)$$

$$\theta = 4^\circ$$

$$x^2 = 98^2 + 80^2 - 2(98)(80)\cos 4^\circ$$

$$x^2 = 362.20$$

$$x = 19.03 \text{ m}$$

Advanced: The police cruiser catches the speeding car when both their distances travelled are equal. Use the formula $d = vt$.

The d of the speeder is 19.03 less than that of the police because of the head start.

$$d_s = v_s t \qquad d_p = v_p t$$

$$d_s = d_p - 19.03$$

$$d_p - 19.03 = 38.9t \qquad d_p = 41.7t$$

$$d_p = 19.03 + 38.9t$$

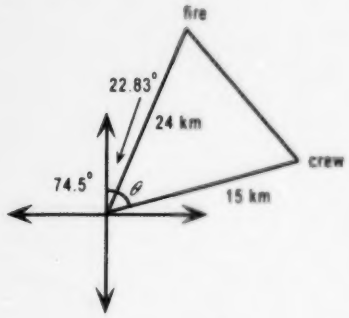
$$19.03 + 38.9t = 41.7t$$

$$19.03 = 2.8t$$

$$t = 6.8 \text{ seconds}$$

Exercise 5 (continued)

6.



a) $\theta = 74.50^\circ - 22.83^\circ$

$\theta = 51.67^\circ$

$x^2 = 24^2 + 14^2 - 2(24)(15)\cos 51.67^\circ$

$x^2 = 354.4633$

$x = 18.33$ km

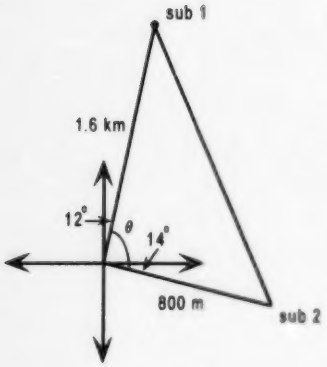
b) $\dfrac{18.83}{3} = 6.275732$ hours

$\dfrac{1\text{ hour}}{60\text{ min}} = \dfrac{0.275732}{x\text{ min}}$

$x = 16.5$ min.

$x = 6$ hours, 16.5 minutes

7.



$\theta = 90^\circ - 12^\circ + 14^\circ$

$\theta = 92^\circ$

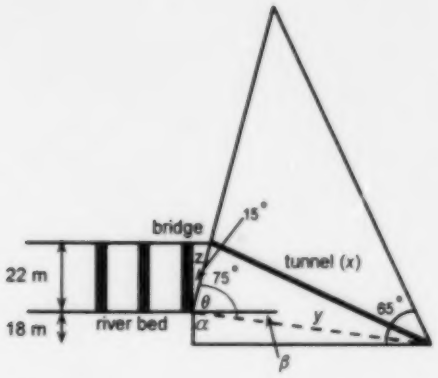
$x^2 = 1.6^2 + 0.8^2 - 2(1.6)(0.8)\cos 92^\circ$

$x^2 = 354.4633$

$x = 18.8$ km

Exercise 5 (continued)

8.



$y^2 = 18^2 + 75^2$

$y = 77.13$

$\cos 15^\circ = \dfrac{22}{z}$

$z = 22.78$

$\tan \alpha = \dfrac{75}{18}$

$\alpha = \tan^{-1}(4.17)$

$\alpha = 76.5^\circ$

$\beta = 90^\circ - 76.5^\circ$

$\beta = 13.5^\circ$

$\theta = 75^\circ + 13.5^\circ$

$\theta = 88.5^\circ$

$x^2 = 22.78^2 + 77.13^2 - 2(22.78)(77.13)\cos 88.5^\circ$

$x^2 = 6375.9784$

$x = 79.8$ m

Volume $= \pi r^2 h$

$= \pi 2^2 (79.8)$

$= 1003.4212 \text{ m}^3$